Analyzing Recreational Boat Acceleration & Boat Coast Down Data: A New Approach

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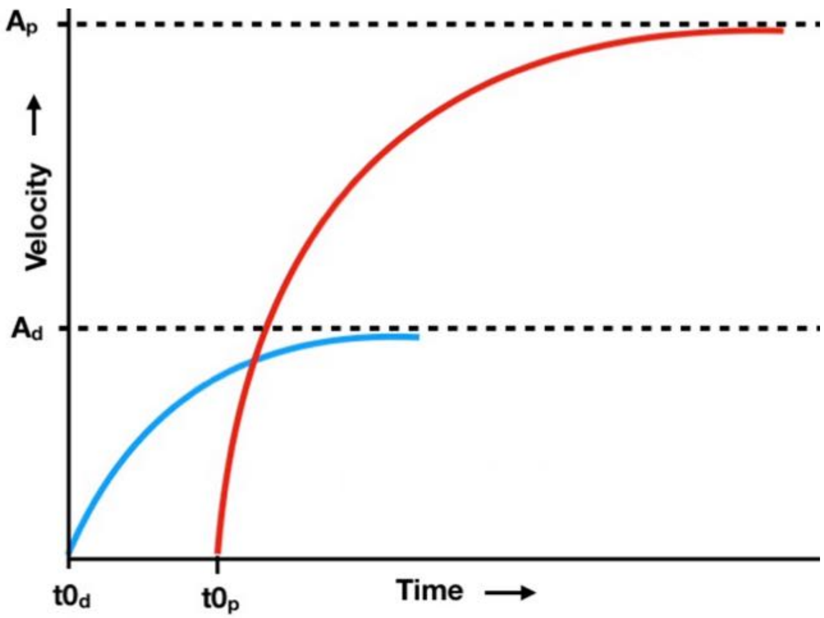

Boat Coast Down testing is sometimes spelled as **Boat Coast-Down testing** or as **Boat Coastdown testing**

Executive Summary

A simple mathematical model for how boats speed up and slow down due to step inputs in throttle position is presented. The model covers boats in displacement mode, transition to planing mode, and in planing mode.

Boat builders and marine drive manufacturers conduct top speed runs in which a boat idling at rest is thrown into forward gear full throttle and ran til it reaches top speed. Elapsed time and velocity are among variables typically recorded multiple times per second during a top speed run. Data output resembles a spreadsheet with each fraction of a second occupying a line in the spreadsheet.

Data output is post processed for things like time to 20 miles per hour, time to 30 miles per hour, and top speed. If data is being collected at ten times per second (10 hertz) ten rows of data are collected every second resulting in quite bit of data, especially if top speed runs are a regular occurrence. Manufacturers compare data between different boats as well as between runs of the same boat with changes, improvements, or updates.

Two similar equations are developed to curve fit the data, one for displacement mode, and another for planing mode. Unlike polynomial curve fits, coefficients in these equations have meaningful significance to the vessel being tested.

The resulting equations are of the nature of

$$\mathbf{V = A\,(1 - e^{-bt})}$$

You may have seen this equation before in relation to charging a capacitor, heat transfer, or charging an air tank.

V = current velocity
A = Maximum velocity
e = an engineering constant, the natural log of 1, approximately 2.718
b = a measure of how quickly the vessel accelerates, you may recognize 1/b as the time constant Tau
t = time

A computer program was written to post process velocity data into distance, propeller slip, acceleration, and more.
Curve fit coefficients were used to calculate propeller thrust at take off.

The equations fit the data very well, allowing just 6 numbers to represent a complete top speed run.

The project also allows boats to be virtually raced against each other or against themselves as changes, improvements, or updates are made. A related future paper involving coast down data is previewed.

Note - Boat Coast Down Testing is sometimes spelled as boat coast-down testing or boat coastdown testing.

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Very Basic Equations and Time Constants

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Introduction

Background Information

An abbreviated version of this report was prepared 30 years ago. Graphs and charts based on processing portions of test data are included to illustrate concepts used to model boat performance data during top speed test runs.

In the early 1990’s I was working at Mercury MerCruiser, the stern drive arm of Mercury Marine, a Brunswick Company. Mercury Marine and MerCruiser used a data recording device they called a PDL (Programmed Data Logger).

PDL data was collected during top speed runs in which the boat was accelerated from zero to top speed.

The PDL recorded a number of variables including elapsed time, engine RPM, and boat velocity ten times a second. Boat velocity was typically picked up from a stern mounted pitot tube. This data was collected prior to wide spread use of inertial navigation systems, widespread availability of consumer GPS devices, and before the U.S. Government stopped degrading GPS signal accuracy.

PDL data was captured in spreadsheet format with several columns of data and a row of data for every tenth of a second. A 25 second run would output about 250 rows of data.

PDL data could be post processed for time to 20 miles per hour, time to 30 miles per hour, for top speed in miles per hour, and propeller slip at top speed.

Some questions were raised? Could top speed run PDL data be post processed to estimate propeller thrust, propeller slip, and other variables? Could coast down data be post processed to estimate propeller thrust? Processing coast down data may be discussed in a future paper, meanwhile a preview of the process is in **Appendix B**.

In today’s time portable data loggers are widely available. One time use systems can be cobbled together very economically. Automotive racing vendors such as VBOX Automotive in the UK provide more durable marine systems.

Note there are complex means of mathematically modeling / estimating / predicting planing boat performance including running scaled models in towing tanks. This paper does NOT predict performance of planing boats in design stages. It demonstrates a means of analyzing performance of a planing boat accelerating from rest to top speed.

We hope viewers will especially find the many graphs and charts in this publication interesting, thought provoking, and useful.

The Initial Problem(s) & Opportunities

Some of these problems and opportunities were recognized up front. Others came to light as the project progressed.

Was it possible to:

1. Use a data logger to estimate propeller thrust?

2. Reduce the amount of space and media required to store top speed run data long term?

3. Store top speed run data more securely from accidental erasure, flood, fire, hackers, and other hazards?

4. Store portable data logger output in a manner a particular data run could be quickly retrieved or regenerated?

5. Use an algorithm to select top speed from data logger output to remove human subjectivity from identifying the vessels's top speed?

   Velocity data bounces around a bit at top speed and it is challenging to repeatedly identify top speed.

6. Meaningfully / visually / graphically compare data from two or more top speed runs side by side?

7. Virtually race two or more vessels against each other using top speed run data?

   Be able to quickly see which vessel is in front and how far they are in front at every moment during a virtual race.

8. Virtually race the same vessel against itself in a different configuration (changes to engine, marine drive, or vessel)?

   The ability to visually see which version of the vessel would be ahead in a race and how far that version would be ahead at every moment during a virtual race. No longer would test crews just be comparing times to twenty mph, times to thirty mph, time to plane, and top speed. They could see both configurations as they raced over the total distance traveled.

9. Use an equation I used about a decade earlier in another setting to differentiate velocity data to calculate acceleration?

I published an internal paper at Charles Machine Works / Ditch Witch[1] on thermal design of mobile hydraulic systems back in 1983. The paper included a mathematical equation representing thermal response of a mobile hydraulic system (such as a trencher) as it warmed up. By design, the hydraulic system on trenchers gradually warms up and levels off at a maximum temperature less than would be hazardous to the system. At maximum oil temperature, a trencher's hydraulic system is emitting as much heat (releasing it to the surroundings) as it is generating.

Trencher hydraulic system thermal curves looked very similarly to velocity vs time curves of a boat accelerating up to top speed at which time the entire force of the propeller is being consumed by drag.

10. Develop a simple mathematical model for a top speed run of a planing boat?

Existing mathematical models for planing boats were and continue to be extremely complex. There was a need for a much simpler model for existing boats in top speed runs and coast downs.

11. Develop an easy to use model of a planing boat for engineers, and boat and marine drive manufacturers that accurately simulates a planing boat's position, velocity, acceleration, and propeller slip during a top speed run?

Such a model might also have application to boats in computer games.[2]

12. Estimate top speed of a boat without actually getting the boat up to top speed?

Boating conditions may be hazardous for top speed runs. Boat operators may not be skilled at high speed operation, those on board may not have helmets required for operating a faster boat, a chase boat may not be available, weather may not cooperate. The lake may not be long enough to reach top speed or be too crowded to safely run at top speed. Yet engineers desperately want to evaluate recent changes to the boat or drive. There is a need to accurately estimate top speed performance from a portion of a high speed run conducted at slower speeds.

13. Estimate propeller slip during during performance runs, not just at top speed?

14. Mathematically or graphically define time to plane?

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[1] CMW Hydraulic Thermal Design Manual. Gary Polson. 21 January 1983.

[2] Chapter 9. Boats and Things That Float. Physics For Game Programers. Grant Palmer. 2005.

15. Graphically understand what happens in transition between displacement mode and planing mode?

16. Model displacement mode and planing mode with similar mathematical functions, using meaningful coefficients for each mode?

17. Use a data logger to estimate drag during coast downs of recreational planing boats?

18. Mathematically investigate passenger stability (impact of velocity, acceleration, and jerk) on seated and standing passengers?

The answer to these 18 questions, at least in some situations, eventually turned out to be yes.

The Project Begins

Many things came together to make this work possible.

As things got underway I teamed up with one of the technicians operating test boats with an interest in my work. This paper will call him Steve.

Steve pointed me toward using a macro in Lotus 1,2,3 (a popular spreadsheet program at that time) to process the data. He showed me how Lotus 123 macros might be able to post process the data, and continued to help during the project.

Eventually, with a lot of help and encouragement from Steve, I developed an approximately 500 line Lotus 1,2,3 macro with many subroutines capable of processing portable data logger output. Among the subroutines included calculating total distance and propeller slip at each time step, producing charts, and able to virtually race two boats against each using previously collected data. The boat racing subroutine even allowed racing the same boat in different configurations against itself.

Integration of raw data is much less noisy than differentiating the same raw data (raw velocity data is integrated to obtain distance and differentiated to obtain acceleration). Velocity was integrated over time to determine total boat travel distance by adding up how far the boat went each tenth of a second and summing those distances over time. Integrating velocity data to obtain displacement usually works fairly smoothly.

Differentiation (determining the slope of the velocity curve over each tenth of a second) typically does not work as smoothly. Instantaneous velocity data is bouncing around meaning the instantaneous slope is bouncing all over the place. However, the derivative (slope) of the velocity curve each tenth of a second was able to be determined by post processing the data using a 9 Point Moving Chord Averaging method[3] I encountered years earlier.

Top Speed Run Velocity Curves Have Two Modes

Top speed run velocity data was graphed and curve fit. Planing boat top speed run velocity curves were found to be composed of two separate curves. Velocity vs time data follows one curve before going on plane, and another curve after going on plane. See **Figure 1**.

Figure 2 shows the same two velocity vs time curves as if they continued on. Meaning as if the displacement curve continued on after the boat began to plane, and the planing curve as if the boat took off in a planing mode.

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[3] Biomechanical Analysis With Cubic Spine Functions. Thomas M. McLaughlin, Charles J. Dillman, and Thomas J. Lardner. The Research Quarterly Vol.48 No.3. October 1977. Page 577.

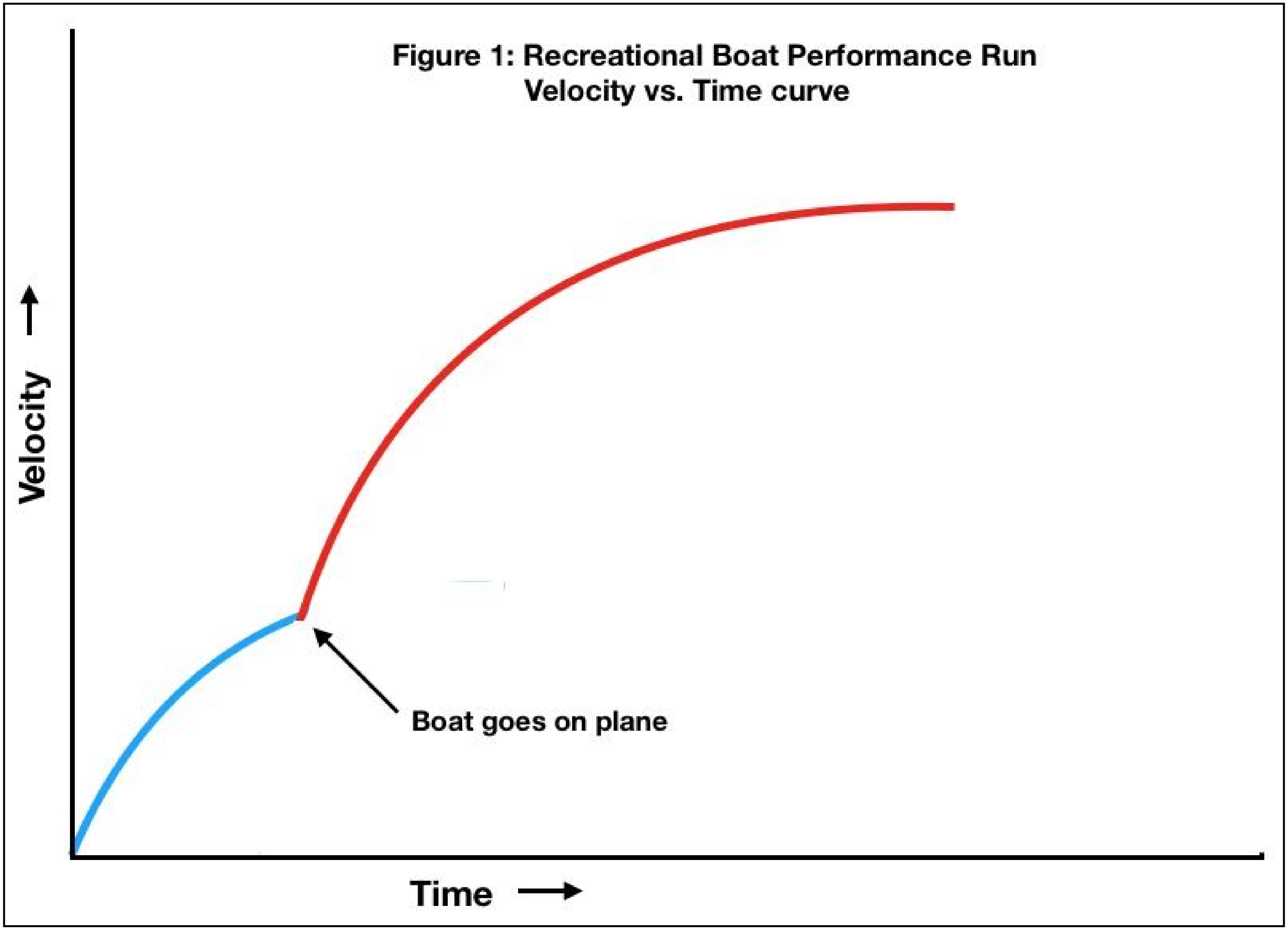


Figure 1: Recreational Boat Performance Run
Velocity vs. Time curve
Velocity
Boat goes on plane
Time

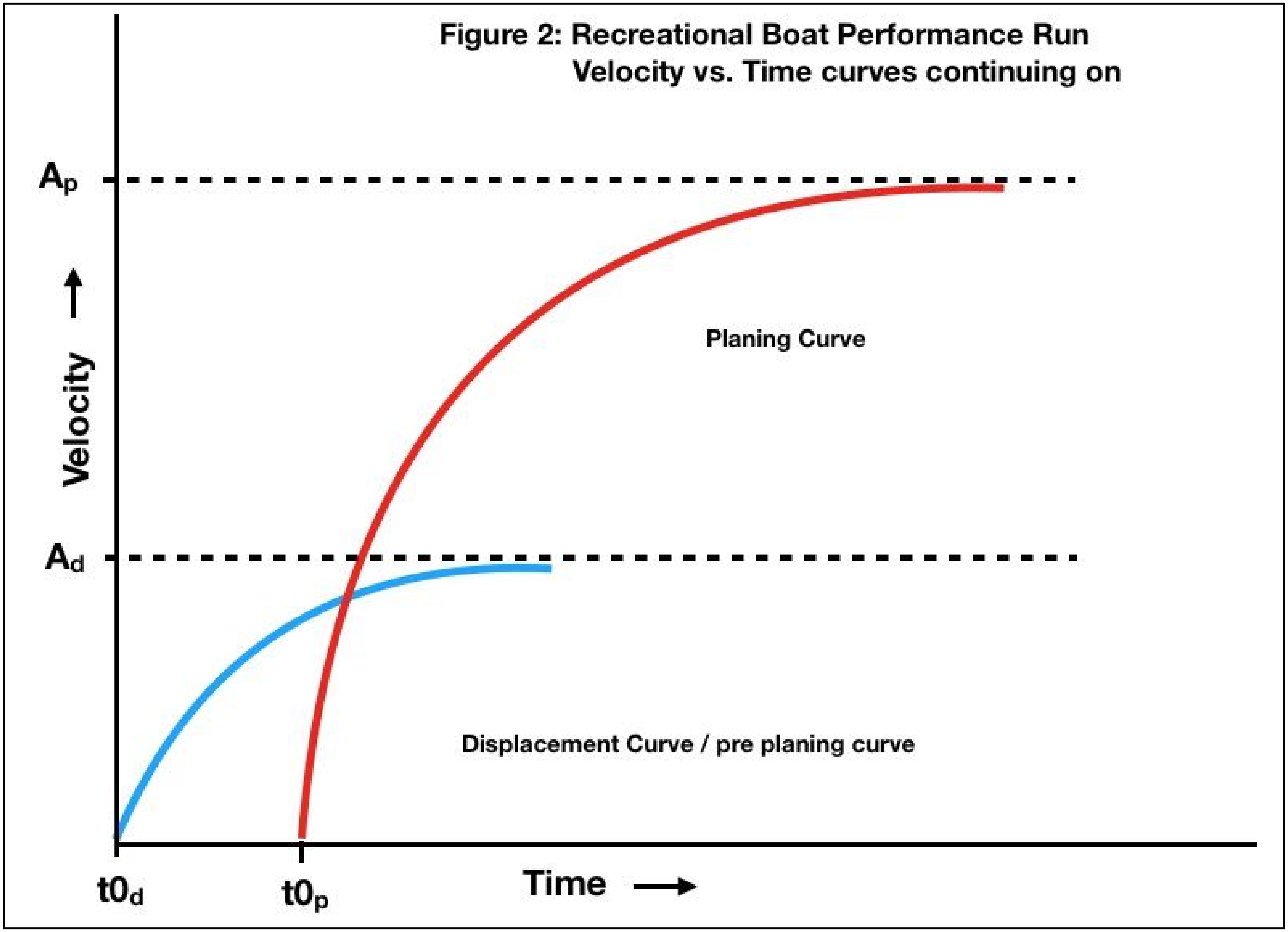


Figure 2: Recreational Boat Performance Run
Velocity vs. Time curves continuing on
Ap
Ad
Velocity
Planing Curve
Displacement Curve / pre planing curve
t0d
t0p
Time

Curve Fitting

The displacement curve (blue curve in **Figure 2**) can be fit with:

$$V_d = A_d\,(1 - e^{-b_d(t - t0_d)})$$

Subscript “d” above refers to the displacement boat portion of the curve (blue curve).

If data collection begins the moment the boat begins to accelerate $t0_d$ would equal zero. Thus in those instances the displacement portion of the performance run (the blue curve) could be curve fit with:

$$V_d = A_d\,(1 - e^{-b_d t})$$

“A_d” is the horizontal asymptote of the blue curve, the velocity the boat would almost reach if it ran in displacement mode forever and never popped up on plane (see **Figure 2**). All that is left is to solve for “b_d” which is a measure of how quickly the curve rises (how fast the boat accelerates). Larger values of “b_d” represent steeper velocity vs time curves. In most real world cases $t0_d$ needed to be defined which is very simple. It is just the time at which the boat begins to take off / accelerate. The macro identified takeoff time and reset the clock to zero for takeoff.

The planing (red curve in **Figure 2**) can be fit with:

$$V_p = A_p\,(1 - e^{-b_p(t - t0_p)})$$

“A_p” is the asymptote of the red curve, the top velocity the boat would almost reach if it ran on plane forever. “$t0_p$” is marked on the time axis. It ($t0_p$) is the time at which the planing curve (red curve) begins if it is extended all the way to the X axis. All that is left is to solve for “b_p” which is a measure of how quickly the planing curve rises (acceleration).

Finding b_d and b_p

The previous section mentioned calculating b_d and b_p. Three methods were used:

1. SigmaPlot. A DOS version of SigmaPlot was used to curve fit data, including identifying $\mathbf{b_d}$, $\mathbf{b_p}$, $\mathbf{A_d}$, and $\mathbf{A_p}$.
2. Their values were guessed. After a while you get a basic feel for their values. You can guess and see what the plot looks like. Then continue to guess as you hone in on the best fit.
3. Calculate $\mathbf{b_d}$ and $\mathbf{b_p}$ from the data as described below.

Values of **A** and **t0** can be visually estimated from the curves. The value of **b** can then be calculated at a few points on the curve (displacement curve or planing curve), and averaged. The value of **b** can also be estimated at certain percentages of top velocity as discussed later in the Time Constant section on **Pages 89-92**.

Representing Performance Run Data With Just 6 Coefficients

$$V_d = A_d\,(1 - e^{-b_d(t-t0_d)})$$

Curve fitting top speed run PDL data allows us to store or represent a few hundred rows of PDL data with just 6 coefficients:

$\mathbf{A_d}$ = velocity asymptote of the displacement portion (non-planing portion) of the run in miles per hour

$\mathbf{A_p}$ = velocity asymptote of the planing portion of the run in miles per hour

$\mathbf{b_d}$ = a measure of how quickly the boat accelerates in non-planing mode

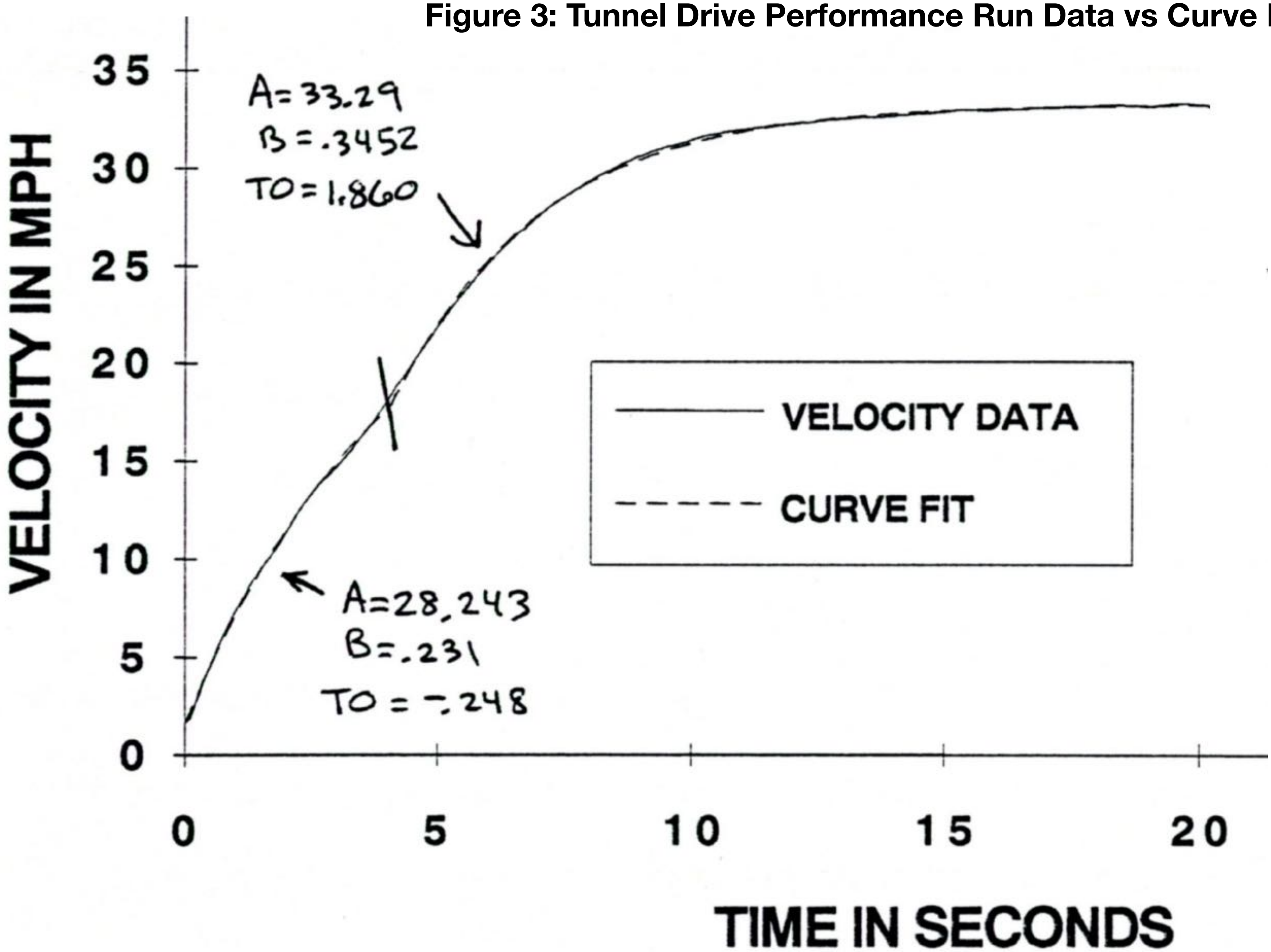
$\mathbf{b_p}$ = a measure of how quickly the boat accelerates in planing mode

$\mathbf{t0_d}$ = time in seconds between zero on the X axis and when the displacement mode velocity curve rises from a value of zero. Sometimes $t0_d$ will be zero.

$\mathbf{t0_p}$ = time in seconds between zero on the X axis and when the planing velocity curve would intersect the X axis if the planing curve was extended.

Figure 3 on the next page charts velocity data from a performance run of a tunnel drive boat, along with a mathematical curve fit of the same data. Additional information about **Figure 3** can be found on **Page 22**.

Figure 3: Tunnel Drive Performance Run Data vs Curve Fit



Curve Fitting Top Speed Run Velocity, Figure 3

The two curves (displacement curve and planing curve) fit remarkably well for the tunnel drive top speed run as seen in **Figure 3**.

Note $T0_d$ has a value of -.248. The negative value is due to the boat starting to accelerate about a quarter of a second before the portable data logger started collecting data.

The non-planing portion of the velocity data in **Figure 3** was curve fit by Sigma Plot as:

$$V = 28.243 \text{ miles per hour} \times (1 - e^{-.231(t+.248)})$$

The planing portion of the velocity data in **Figure 3** was curve fit by:

$$V = 33.29 \text{ miles per hour} \times (1 - e^{-.3452(t-1.860)})$$

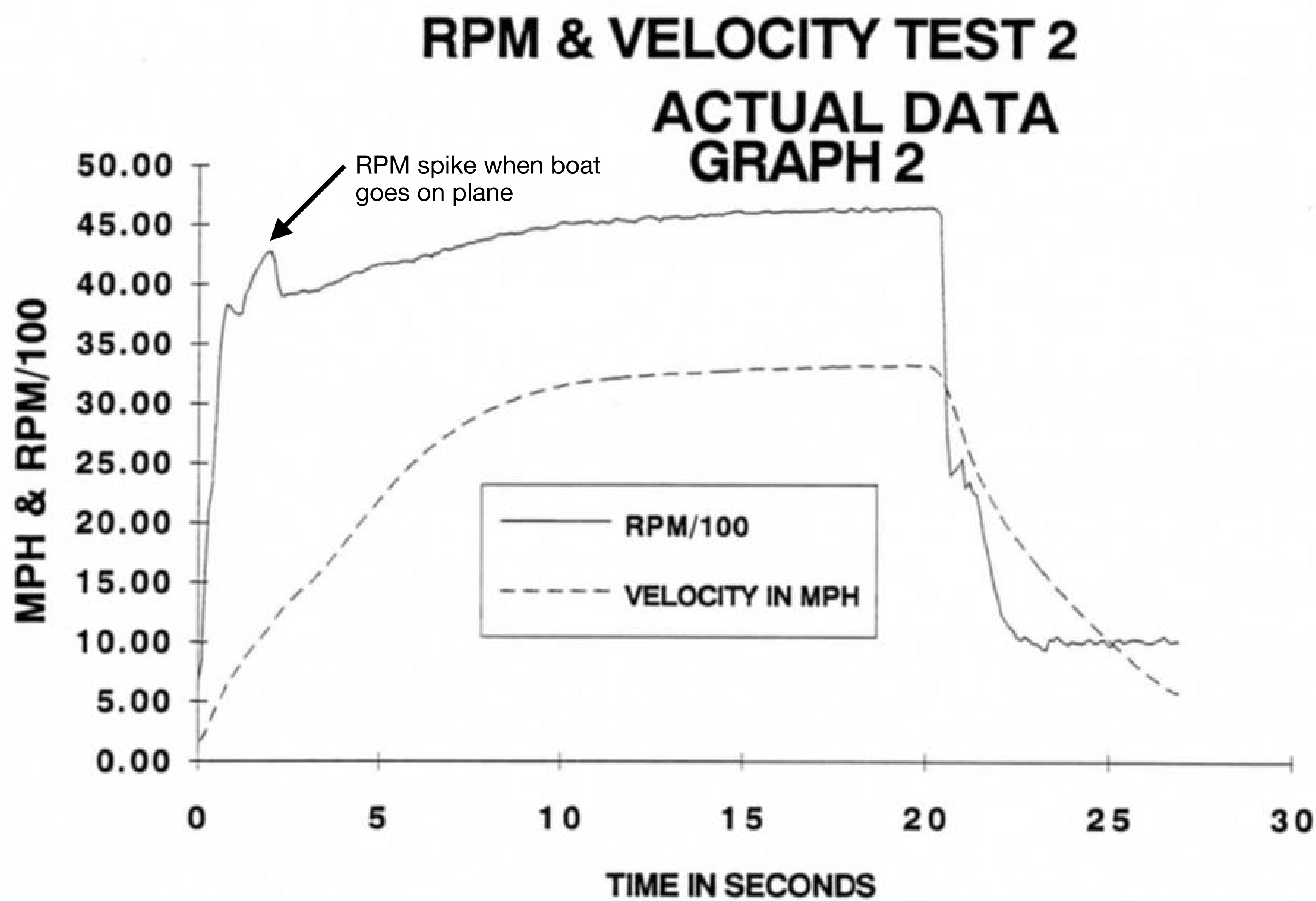
RPM & Time Curve Figure 4

Figure 4 is an example of an RPM vs Time curve combined with a Velocity vs Time curve.

Figure 4 shows how RPM rapidly increases from take-off, then gradually climbs as top speed is approached.

It is interesting to be able to see the spike in RPM in **Figure 4** about two to three seconds into the run when the boat changes from displacement mode to planing mode as the boat accelerates up to top speed.

Figure 4 : RPM & Velocity



The Six Coefficients Provide Some Other Opportunities:

1. In the past it was challenging to consistently identify top speed of faster boats from reams of data logger output because speed data bounced around ten times a second. Now the asymptote for top speed in planing mode is impartially determined by Sigma Plot.

2. Sometimes only partial data was available from a top speed run. Just a small bit of data from the planing section of a performance run can be curve fit for the value of "$\mathbf{A_p}$" (top speed). This approach was tested on performance runs of 70 plus mph boats. The method predicted top speeds a couple miles per hour too fast perhaps due to wind drag becoming a greater issue at these speeds. This method could allow estimating top speed of performance boats without actually running them at top speed in a lake of limited length or with a driver not capable of safely handling the boat at extreme speeds. The boat just needs to be accelerated hard to a midrange planing speed to provide the data needed for curve fitting.

3. Thrust at takeoff can now be calculated. When the boat takes off the boat has no drag. All thrust goes into accelerating the mass of the boat. Force (which is thrust) = Mass times Acceleration. As will soon be seen, take off thrust can be estimated from weight of the boat and the six coefficients. See added mass comment on **Page 81**.

Measuring / Estimating Marine Propeller Thrust

In the 1990s recreational planing boat propeller thrust was measured / estimated by:

1. Use of complex instrumentation and one or more compression plates, in combination with slip rings

2. As part of developing the Blackhawk drive, Brunswick / Mercury Marine patented[4] a means to monitor thrust on each propeller of a counter rotating surfacing drive. See **Figure 5**.

3. Comparing acceleration sensed during runs of different vessels

4. Bollard pull tests

Bollard pull tests tie a boat to a solid object such as a boat dock using a rope and measure force (tension) in the middle of the rope as the boat pulls away from the dock at various RPMs. While bollard pull testing does provide thrust data, it may not be representative of top speed run thrust, especially when the boat is on plane.

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[4] U.S. Patent 5,527,194. Thrust Sensor for Marine Drives. Inventors: William A. Strong, Gerald E. Weitz, John W. Bahara. Assigned to Brunswick. Patent issued June 18, 1996.

United States Patent [19]
Strong et al.

[11] **Patent Number: 5,527,194**
[45] **Date of Patent: Jun. 18, 1996**

[54] **THRUST SENSOR FOR MARINE DRIVES**

[75] Inventors: **William A. Strong**; **Gerald E. Weitz**; **John W. Behara**, all of Stillwater, Okla.

[73] Assignee: **Brunswick Corporation**, Lake Forest, Ill.

[21] Appl. No.: **191,272**

[22] Filed: **Feb. 2, 1994**

[51] **Int. Cl.**^{6} **B63H 5/10**
[52] **U.S. Cl.** **440/80**; 440/900; 73/862.049
[58] **Field of Search** 73/862.49; 440/79–81, 440/83, 76, 88, 900, 113; 384/609

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Attorney, Agent, or Firm—Andrus, Sceales, Starke & Sawall

[57] **ABSTRACT**

In a marine drive (**10**), a nonrotational thrust sensing device (**202** or **204**) is placed between the stationary housing (**26'**) and rotating propeller shaft (**40'** or **42'**) to measure the thrust output of a propeller (**12** or **14**) by measuring the equal and opposite force applied to the housing by the propeller shaft (**40'** or **42'**). A thrust bearing (**214** or **248**) is situated between the rotating shaft and the nonrotating thrust sensing device. A thrust ring (**246**) and adapter (**250**) provide adoption of the thrust sensing device to a particular housing and shaft configuration. In a marine drive having counter rotating coaxial propeller shafts, first and second thrust sensors individually sense the thrust of inner and outer propeller shafts, respectively, to provide separate sensed thrust for each.

17 Claims, 5 Drawing Sheets

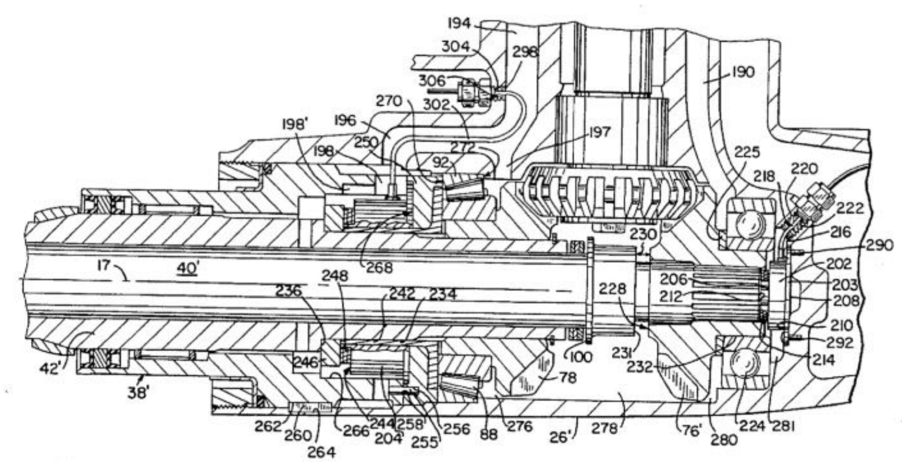


Figure 5: Brunswick U.S. Patent 5,527,194.
Thrust Sensor for Marine Drives

A New Way to Estimate Thrust at Takeoff

At the moment of takeoff in a top speed run, every bit of propeller thrust goes into accelerating the boat. Once the boat gets up and going and continues to accelerate, a portion of the thrust goes toward overcoming drag, and a portion goes toward continuing to accelerate the boat.

Therefore, take off thrust can be estimated from Force = Mass X Acceleration

If you know the acceleration of a planing boat at takeoff AND its Mass you can estimate thrust at time equals zero, which is Force in the equation above.

Mass is easily estimated by weighing the boat with all the gear and the amount of fuel normally loaded for testing, then adding the weight of those onboard.

Acceleration is the slope of the Velocity vs Time curve.

Five ways / methods to estimate slope of the Velocity vs Time curve at takeoff are listed below:

1. Graphically - plot Velocity vs Time data of the boat performance run and use a protractor to estimate its initial slope,

2. Calculate slope - use the data during the first few tenths of a second after take-off to calculate slope.

3. Method #2 above except an algorithm is used to smooth velocity data during the process of calculating slope during the first few tenths of a second of the Velocity vs Time curve.

4. Derivatives - (A) mathematically curve fit the displacement portion of the takeoff curve (see **Page 18**), (B) mathematically take the derivative of that equation (see the next section), (C) evaluate that derivative at take-off time to determine slope of the velocity vs time curve at takeoff.

5. Use Tau, the time constant (see **Figure 35** and **Figure 36**) and maximum velocity. b = 1/Tau. Initial slope = Maximum Velocity / Tau for each mode (displacement mode and planing mode).

This paper will focus on Method #4 above.

Using Derivatives to Estimate Propeller Thrust at Takeoff

F= Mass X Acceleration

Weigh the boat and those on board to estimate mass. If we can determine acceleration at take-off we can determine force, which is thrust.

Slope of the Velocity vs Time curve at take-off time equals acceleration.

As seen on **Page 7** the generic form of the equation below is used to curve fit the displacement portion of top speed runs:

$$V_d = A_d\,(1 - e^{-b_d(t-t0_d)})$$

$$\text{acceleration} = dv_d/dt = A_d(0 - (-b_d)e^{-b_d(t-t0_d)}) = A_d b_d e^{-b_d(t-t0_d)}$$

evaluate derivative above at time = $t0_d$ to determine initial slope

$$(dv_d/dt)\,| = A_d b_d e^{-b_d(0)} = A_d b_d$$

Slope of the velocity curve at takeoff which is time = $t0_d$ is simply $\mathbf{A_d}$ times $\mathbf{b_d}$

More simply, acceleration is the asymptote for top speed of the boat in displacement mode times a measure of how quickly the boat accelerates.

Calculating Propeller Thrust Using the Derivative Method: an Example

Values below come from the outboard motor powered tunnel boat tested in **Figure 3**.

Per the derivative method developed on **Page 27**,
Acceleration at takeoff in **Figure 3** can now be estimated as **A** times **b**.

A = 28.243 miles per hour

b = .231 / seconds

acceleration = $A_d \times b_d$ = (28.243 miles/hr) × (.231 / seconds) × (5280 ft/mile) × (1 hour / 3600 seconds)

acceleration at takeoff = 9.56 ft/sec^2

Thrust can now be calculated from a boat weight of 2475 pounds including those on board during testing.

Boat Mass = 2475 pounds / 32.2 = 76.86 slugs

Thrust at takeoff = Mass × acceleration

Thrust at takeoff = 76.86 slugs × 9.56 ft/sec^2

Thrust at takeoff = 735 pounds

Due to some simplifications of this model, thrust at takeoff in **Figure 3** may not actually be 735 pounds. However the model can be used to compare multiple runs.

The Need to Differentiate Velocity Data

As mentioned earlier, Velocity vs Time data was curve fit, then mathematically differentiated to determine acceleration at time = 0.

This section focuses on another method of instantaneously determining slope of the velocity curve (acceleration) without first having to mathematically curve fit the data.

Basically, we want to estimate slope of the velocity curve over each tenth of the top speed run.

It is challenging to accurately differentiate raw data one step at a time. Instantaneous slopes of raw data such as velocity curves during acceleration are often very noisy. While total velocity quickly increases, actual change in velocity over each tenth of a second as measured bounces around quite a bit (contains noise).

As expected, when slope was directly calculated every tenth of a second from boat velocity data as recorded by the data logger, results were very noisy. The slope curve (acceleration) was even noisy when velocity data was smoothed by a moving three point average before calculating slope. The acceleration curve was much smoother when a Nine-Point Moving Chord Average was used to calculate the slope of the velocity curve at each point.

I encountered the Nine-Point Moving Chord Average when weight lifters were being analyzed by film (vertical position of the bar), then differentiated twice to determine acceleration in an effort to determine the actual force of the bar. I spoke with Mr. McLaughlin, one of the authors of a related technical paper[5] back in the early 1980s.

Applying the Nine-Point Moving Chord Average method to velocity data allowed estimating acceleration once underway. However the method requires a few data points before it begins to return results, thus it does not actually supply acceleration at takeoff. The 9 Point Moving Chord Average method uses a couple data points from when the boat was almost at rest before it took off. Otherwise, it does supply acceleration data for about half a second later which can be compared with the value obtained by differentiation method at takeoff.

[5] Biomechanical Analysis With Cubic Spine Functions. Thomas M. McLaughlin, Charles J. Dillman, and Thomas J. Lardner. The Research Quarterly Vol.48 No.3. October 1977. Page 577.

9-Point Moving Chord Average Method

The Nine-Point Moving Chord Average formula which was originally written to calculate velocity from displacement data is below:

$$V_n = (X_{n+4} + X_{n+3} - X_{n-4} - X_{n-3}) / (14 \times \Delta t)$$

where **V** is velocity, **X** is displacement, **n** is the data point, and **Δt** is the time interval.

In this paper, we already have velocity data and are trying to estimate acceleration (slope of the velocity curve), so our Nine-Point Chord Average equation became:

$$A_n = (V_{n+4} + V_{n+3} - V_{n-4} - V_{n-3}) / (14 \times \Delta t)$$

where **A** is acceleration, **V** is velocity, **n** is the data point, and **Δt** is the time interval.

The 9-Point Moving Chord Average method adds velocity data three and four time steps in front of you and subtracts velocity data three and four time steps behind you, then divides the result by fourteen times the size of the time step. In this report, the time step was one-tenth of a second.

Visually Comparing Both Methods (Derivative vs 9 Point Moving Chord Average Method)

Figure 6 charts acceleration during a top speed run using both methods. The curve represented by the line represents the 9 Point Moving Chord Average Method. The curve using small squares to represent each data point represents the mathematical calculation of the value of the derivative of the fitted curve at that point.

Both methods provide very similar values.

Disregard data to the right of 20 seconds into the run. That portion of the data was collected as part of a study of Coast Down data which will be introduced in **Appendix B**.

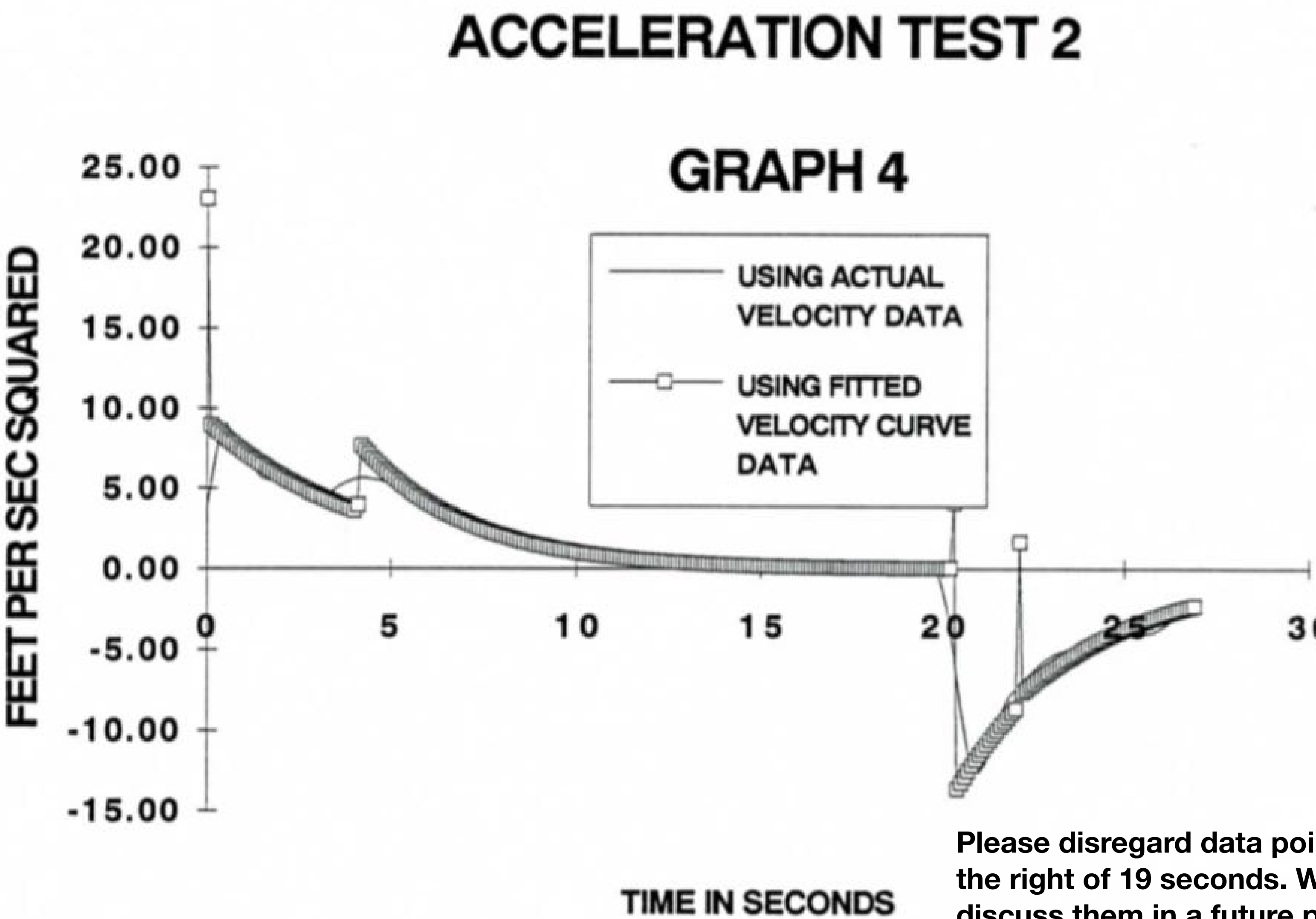
Of note is the way the “line curve” keeps leaving or arriving about 5 time steps before or after the the differential curve. That is due to the 9 Point Moving Chord Average Method requiring 5 data points before it begins to return values.

The break in the curves at about 4 seconds is caused by the boat going on plane. It begins to accelerate faster when it pops up on plane.

After about 13 seconds the boat is running near top speed and acceleration is approaching zero.

The acceleration curves in **Figure 6** are thought to be from for the Top Speed run seen in **Figure 3**.

Figure 6: Comparing Acceleration Calculation Methods



Please disregard data points to the right of 19 seconds. We will discuss them in a future paper.

Acceleration Chart Comments, Figure 6

Time to Plane time (how long it takes to get the boat up on plane) is far more obvious on an Acceleration vs Time chart compared to on a Velocity vs Time Chart.

In the Velocity vs Time Chart (**Figure 3**), planing time is seen as a slight point of inflection in the curve at about 4 seconds into the run.

In the Acceleration vs Time Chart (**Figure 6**), planing time is a discontinuity in the curve about 4 seconds into the run. It is very obvious when the boat goes on plane.

Before someone wants to calculate planing time to a fraction of a second, please view the reference on lag time in pitot tube velocity measurements discussion associated with **Figure 7** and the transition zone discussion associated with **Figure 29** and **Figure 30**.

Acceleration During a Top Speed Run, Figure 6

During a Top Speed run Acceleration quickly increases to maximum value as the boat begins to take off, then deceases as the boat continues gaining speed while still in displacement mode.

Acceleration increases once more as the boat goes on plane and accelerates on up to top speed. Near the end of the run, Acceleration approaches zero as the boat levels out at top speed. See **Figure 6**.

Components of Thrust

As mentioned earlier, at takeoff for a top speed run of a planing boat, all thrust goes into accelerating the boat. Once the boat gets up and running some thrust goes into accelerating the boat and some thrust goes into overcoming drag.

As the boat planes and approaches top speed, almost all thrust goes into overcoming drag and just a little thrust goes into accelerating the boat. That, along with trying to find optimal trim for top speed is why it can take a long time to reach maximum velocity.

At maximum velocity (top speed), all thrust goes into overcoming drag, or Thrust = Drag.

Accuracy of Pitot Tube Velocity Data

Over the course of this project and the associated Coast Down work, some data right after an impulse (takeoff or coast down) did not seem logical. Velocity data was more closely examined.

Boat pitot tube based speedometers (see **Figure 7**) consist of a pitot, a device bolted to the lower rear of the transom with a forward facing hole. The hole communicates the pressure up the transom, then upwards to a fitting allowing tubing to run from there to the pitot based speedometer at the helm. Pitot based speedometers use a diaphragm. The diaphragm has pitot tube pressure on one side and static pressure (atmospheric pressure) on the other side. A flexible diaphragm detects the difference between those two pressures, the speedometer converts pressure difference to speed.

A YouTube installation video[6] does a nice job of showing the pitot tube and how it connects to the system.

MerCruiser Engineering Testing used a suction cup to stick a more accurate pitot tube on vessel transoms during testing and were able to stick the same tube on every vessel. The tubing with the water ran to an instrumented suitcase where velocity was recorded. While Pitot Tube velocity data appeared to be accurate for determining top speeds, the response pitot tube velocity data under hard acceleration and deceleration was questioned. Several instances / anomalies indicated pitot tubes failed to instantly respond to rapid changes in boat velocity.

A text book reference[7] is representative of several discussions on this topic. For fast response in liquids, systems need large diameter tubes and short lengths, neither of which were present in the pitot tubes being used.

Sanshin (Yamaha) recognized this problem in U.S. Patent 4,956,977.[8] Yamaha developed a speedometer system with greater accuracy during abrupt speed changes.

This delay is likely why data points do not always line up on the same exact time under hard acceleration or hard deceleration. There will be more on this topic in examples provided later in this paper.

---

[6] How to Fit Your Boats Speedometer Pitot Tube. Big Island MN. 19 April 2017. YouTube.

[7] *Mechanical Measurements* by T.G. Beckwith and N.. Lewis Buck. 1969. Pages 398-403 (Section 13.11 Dynamic Characteristics of Pressure-Measurement Systems). This reference investigates the dynamic response of pitot tubes detecting pressure levels in liquids in detail.

[8] U.S. Patent 4,956,977. Vessel Speed Detecting Device. Assigned to Sanshin (Yamaha). Provides a more accurate speed signal during abrupt vessel speed changes. Uses a pressure sensor, a digital conversion map, and a pressure/speed calculator to modify output of the pressure transducer during abrupt speed changes.

Figure 7: SeaStar Solutions Pitot Tube

What We Have Learned So Far

Planing boat top speed runs can be represented with two curves. One curve for before the boat gets up on plane and one curve for after the boat gets up on plane. See **Figure 1**.

Those two curves are equations of the form of

$$V = A\,(1 - e^{-bt})$$

Where V= velocity, A= top speed, e= mathematical constant representing the natural logarithm of 1, b= a constant representing how quickly the boat accelerates, and t=time

Using these two equations, thousands of pieces of data from a top speed run can be reduced to 6 coefficients.

Due to the mathematics behind these equations, thrust at takeoff = **A X b**

Velocity vs time data could be collected from a portion of a top speed run, curve fit, and used to forecast the rest of the run, including top speed and propeller slip.

The 9-Point Chord Average Method was used to smooth acceleration data.

Pitot tube velocities were suspected of not responding quickly enough to provide instantaneous velocity data during the most dynamic portions of a top speed run. Pitot tubes were very capable of providing top speed data because speed was not rapidly changing during that portion of the run.

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A Full Series of Charts From a Top Speed Run

Excel Full Series of Charts From a Top Speed Run

To produce this series of charts, a portable data logger captured data during a top speed run, that data was post processed by a Lotus123 macro. The resulting spreadsheet was saved and imported into Libre Office Calc and saved as an Excel file, Excel was then used to create the charts.

The data was exported to Excel in order to produce the charts in a larger format than in Lotus 123.

A total of 11 charts were produced. They will be presented and discussed a few at a time.

Some Details of This Specific Run

CRS drive - a counter rotating surfacing drive

Drive Gear Ratio: 1.5:1

Propellers 24 and 26 pitch propellers (the smaller one up front and the bigger one behind)

Boat was a 23 foot performance boat

These charts are for illustrating concepts presented in this paper, not for disclosing the performance of some exact boat and drive combination. Thus some vessel and drive specifications will not be disclosed.

RPM, Velocity, and Acceleration Curves, Figure #8, Figure #9, & Figure 10

The Velocity vs Time curve exhibits a point of inflection at about 3.3 seconds and another less obvious one at about 4.3 seconds. Note when you mouse over a data point on the actual chart in Excel the X and Y values are displayed making it easy to identify inflection points.

Similarly, the RPM vs Time curve exhibits a point of inflection at about 4.3 seconds. Note this is about 1 second after the first point of inflection observed in the Velocity vs Time curve.

Besides just visually looking at the curves and mousing over the area near inflection points, this report also used three separate procedures to estimate acceleration. The results of two of those methods are plotted. The 9-Point Moving Chord Method shows the first point of inflection in the velocity curve as the bottom of the vertical line at about 3.6 seconds and the second point of inflection as the top of the vertical line at about 4.6 seconds (see **Figure 9** & **Figure 10**). The 9-Point Moving Chord Method smooths data and does not quickly respond to large inputs, thus these times are a little later than those observed on the Velocity Chart.

Similarly the inflection point on the RPM curve can be seen on the RPM Jump curve (**Figure 17**). Change in RPM each tenth of a second was calculated to detect when the boat actually takes off. The Lotus 123 Macro searched for two simultaneous jumps of 40 RPM or greater to make sure the top speed run had began, see **Figure 18**.

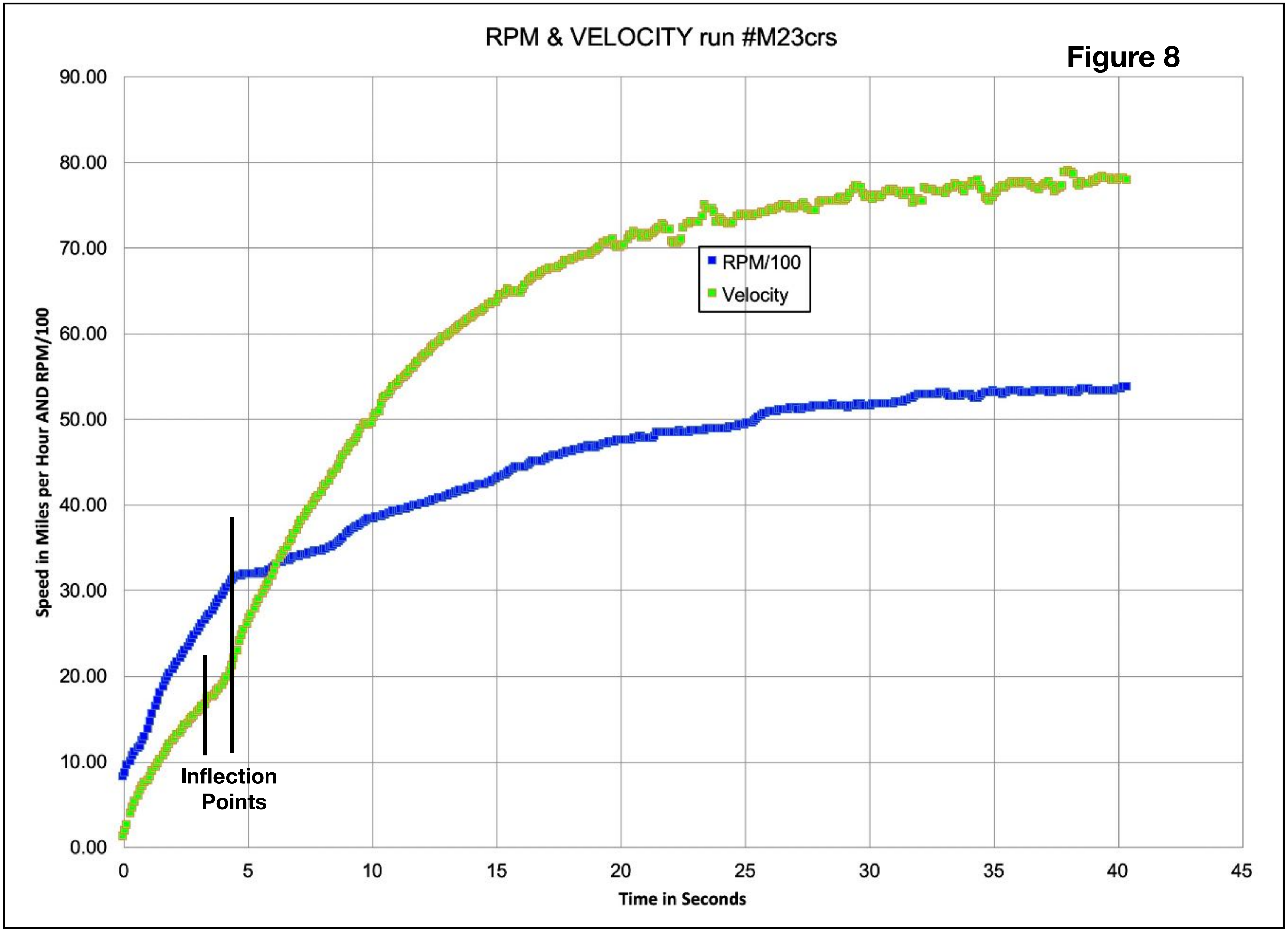
The RPM Jump Curve shows RPM Jump begins to dive at 4.2 seconds. Although RPM continues to rise for many seconds after 4.2 seconds, it rises much slower.

At these inflection points on the Velocity vs Time Curve (3.3 seconds and 4.3 seconds) the boat is either rising up on plane, the large submerged propellers are starting to surface, or both. The props may begin to break free of the surface of the water at about 3.3 seconds and the boat goes on plane at about 4.3 seconds.

The bend at 4.3 seconds can also be observed on the Propeller Slip vs Time Curve presented later as **Figure 14**.

From the Velocity Curve you can see this is a performance boat with a top speed approaching 80 miles per hour.

You can also see how hard it is to get two large counter rotating propellers up and spinning fast. At about 3150 RPM the RPM curve begins to flatten. It does continue to rise, but you can see it almost looks like it is a struggle for the engine to continue to wind up. This can also be observed in the RPM Jump Curve presented later as **Figure 17**. Small jumps in the RPM and Velocity curves past about 25 seconds are at least partially due to the operator searching for the optimum trim to get the boat to maximum speed.


RPM & VELOCITY run #M23crs
Figure 8
Speed in Miles per Hour AND RPM/100
Time in Seconds
RPM/100
Velocity
Inflection Points
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
0
5
10
15
20
25
30
35
40
45

9-Point Moving Chord Acceleration Curve, Figure 9

Velocity data is picked up from the pitot tube speedometer. The 9-Point Moving Chord Average process was used to smooth acceleration as calculated from point to point velocity data on **Pages 29, 30, and 31**. Slope of the velocity data over each tenth of a second time step represents acceleration. However, direct calculation acceleration results in some noise, especially in certain portions of the curve.

Peak acceleration was about 12 ft per second squared, or about 1/3 g. When counter rotating surfacing drives takes off with two large propellers fully submerged. It is definitely not like launching a rocket.

The bottom of the first trough in the acceleration curve occurs at about 3.6 seconds. Between four and five seconds the boat comes up on plane (note the nearly vertical acceleration curve), then acceleration gradually decreases as maximum speed is approached.

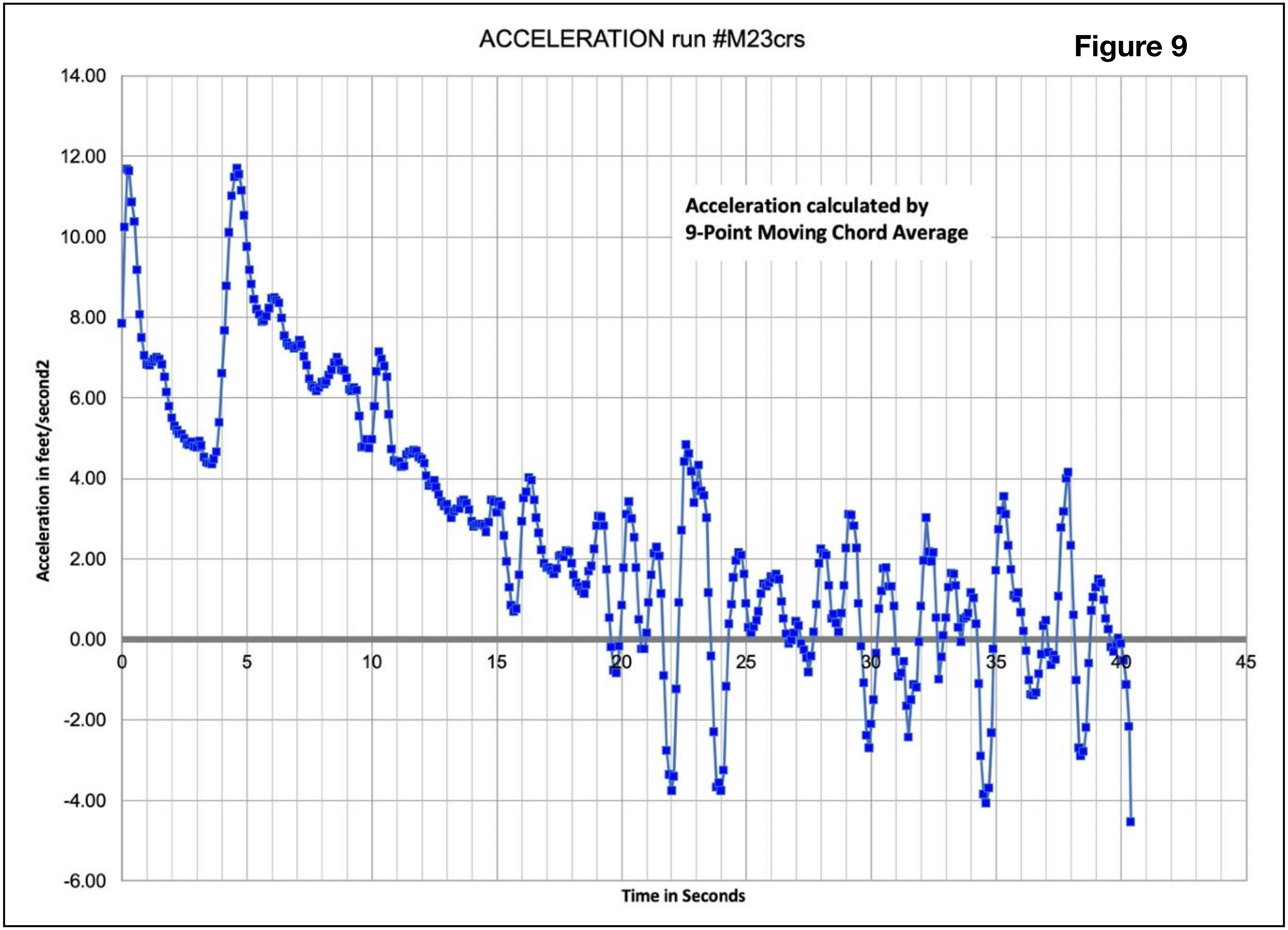
Oscillations beginning at about 20 seconds into the run were due to the operator trimming the vessel hunting for top speed and to the boat bouncing around creating slight changes in velocity as the boat approached top speed.

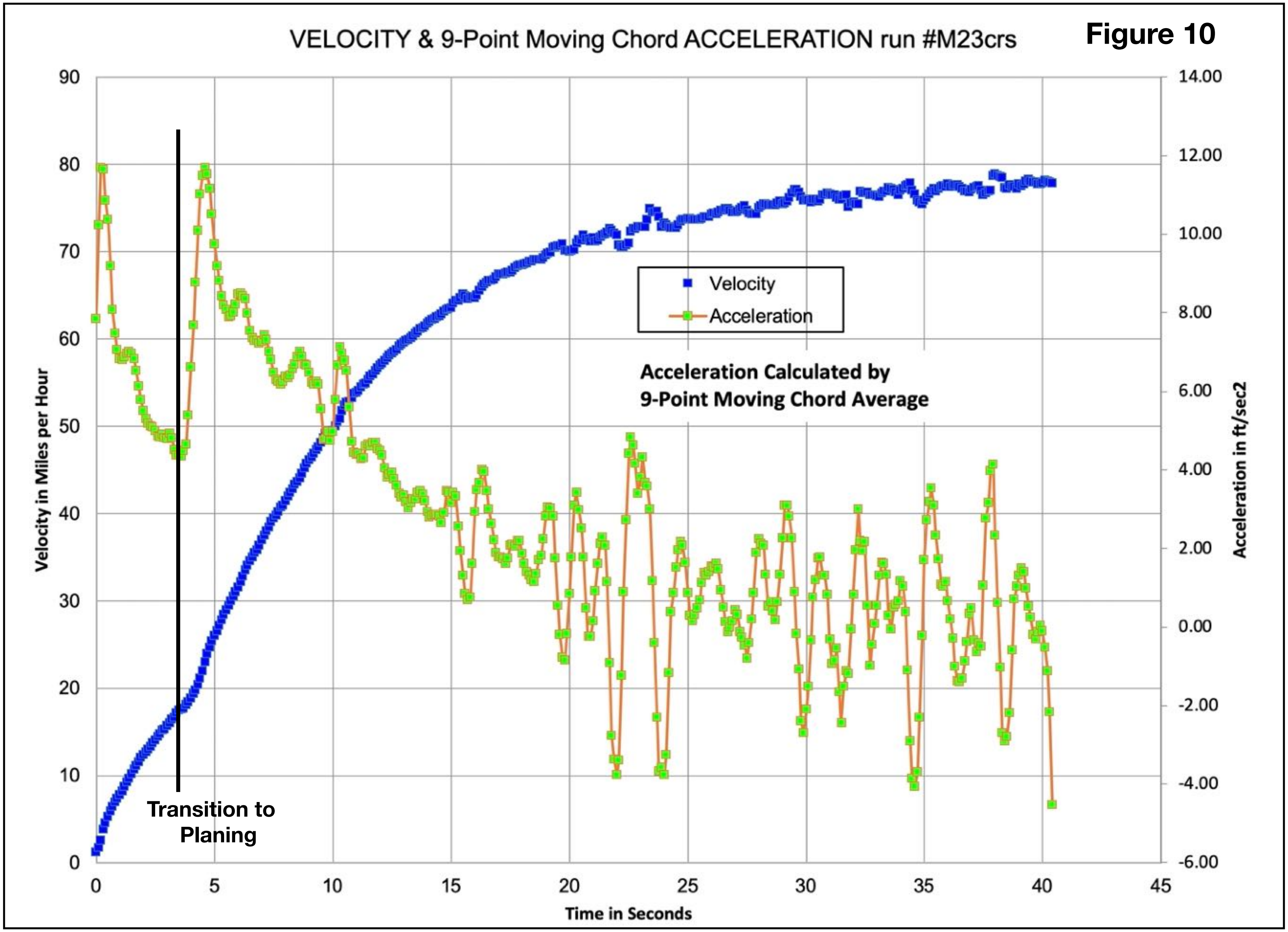
Almost all propeller thrust is being consumed in overcoming various forms of drag (water drag, air drag, etc.) as the boat approaches top speed.

Velocity & 9-Point Moving Chord Acceleration Curve, Figure 10

Several points made earlier can now been seen simultaneously as Velocity and Acceleration are displayed in **Figure 10**. For example inflection point in the velocity curve at about 4 seconds and 18 miles per hour represents the boat going on plane. Looking up from that infection point to the acceleration curve a discontinuity in acceleration is observed as acceleration was decreasing before the inflection point in the velocity curve, then rapidly increased as the boat goes on plane (less drag so same force accelerates the boat at a higher rate).

In **Figure 10** the velocity curve begins to flatten as it approaches top speed, while the acceleration curve begins to bounce a little because the boat is still continuing to pick up a little speed.


ACCELERATION run #M23crs
Figure 9
Acceleration calculated by
9-Point Moving Chord Average
Acceleration in feet/second2
Time in Seconds
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
0
5
10
15
20
25
30
35
40
45


VELOCITY & 9-Point Moving Chord ACCELERATION run #M23crs
Figure 10
Velocity
Acceleration
Acceleration Calculated by
9-Point Moving Chord Average
Transition to
Planing
Velocity in Miles per Hour
Acceleration in ft/sec2
Time in Seconds
0
10
20
30
40
50
60
70
80
90
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
0
5
10
15
20
25
30
35
40
45

Acceleration Comparison Curve, Figure 11

The Acceleration Comparison Curve in blue in **Figure 11** shows why the 9-Point Moving Chord Averaging method was applied to Acceleration data. The Non-Smoothed Acceleration Curve (in orange) obviously has more noise. Noise is more visible at both ends of the curve. Non-Smoothed Acceleration data is noisy shortly after takeoff because velocity data is bouncing around quite a bit. Then similarly, out near reaching top speed, velocity data is bouncing around more wildly as the operator is trimming the boat for top speed and the boat itself is bouncing around a bit.

Especially after about 22 seconds into the run, it becomes obvious how much 9-Point Moving Chord method can smooth data. In this instance it is smoothing the acceleration curve calculated directly from velocity data. The blue curve is fairly smooth while the orange dots are bouncing around all over the place.

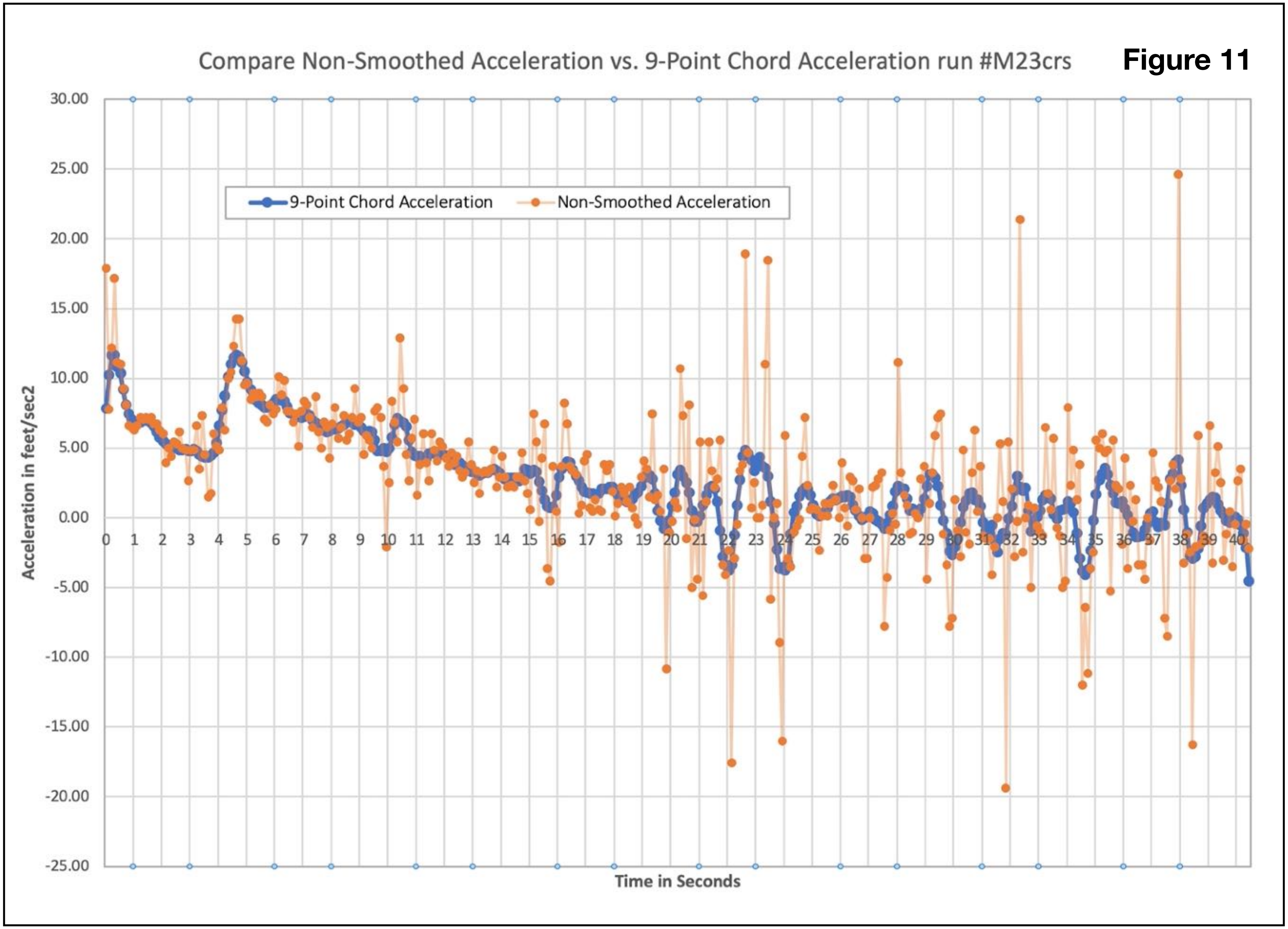
Distance Curve, Figure 12

It does not get any simpler than the distance versus time curve, **Figure 12**. Distance traveled each tenth of a second is calculated as how far the boat would have traveled at the end of that interval. A running total of those distances provides distance.

Distance data comes into play in the Virtual Boat Race section and in calculating propeller slip.

Twenty & Thirty Miles Per Hour Times Curve, Figure 13

Takeoff time was identified (**Figure 18**). Time was counted from take off to the first time velocity reached or exceeded 20 miles per hour and first time velocity reached or exceeded 30 miles per hour. Those times are shown in **Figure 13**.


Compare Non-Smoothed Acceleration vs. 9-Point Chord Acceleration run #M23crs
Figure 11
9-Point Chord Acceleration
Non-Smoothed Acceleration
Acceleration in feet/sec2
Time in Seconds
30.00
25.00
20.00
15.00
10.00
5.00
0.00
-5.00
-10.00
-15.00
-20.00
-25.00
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40

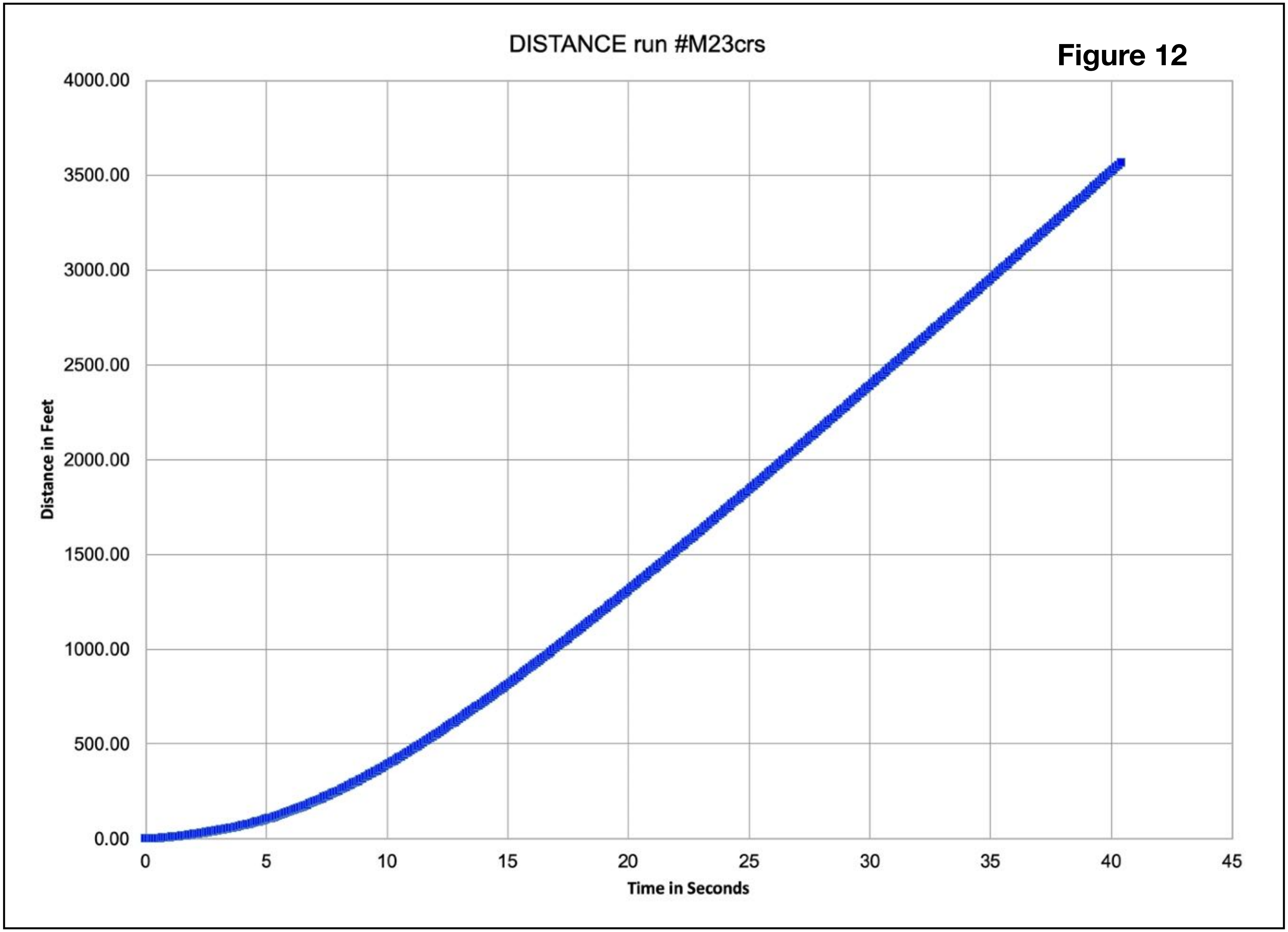
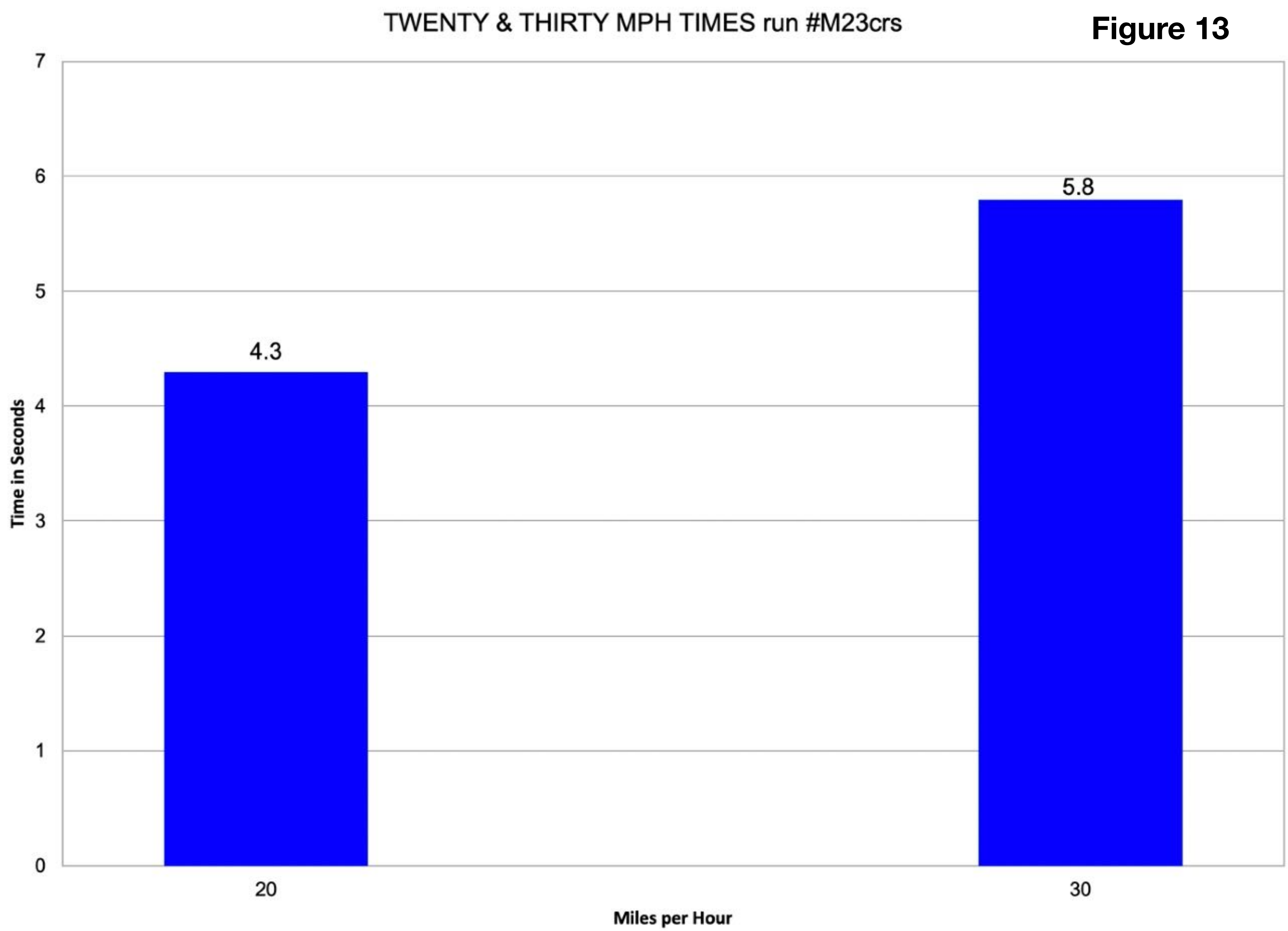

DISTANCE run #M23crs
Figure 12
Distance in Feet
4000.00
3500.00
3000.00
2500.00
2000.00
1500.00
1000.00
500.00
0.00
0
5
10
15
20
25
30
35
40
45
Time in Seconds

Figure 13


TWENTY & THIRTY MPH TIMES run #M23crs
Time in Seconds
7
6
5
4
3
2
1
0
4.3
5.8
20
30
Miles per Hour

Prop Slip vs Time Chart, Figure 14

The Propeller Slip curve, **Figure 14**, was developed by comparing displacement data to how far the boat should be moving each time the propeller revolves with how far it actually went. The 1.5:1 gear reduction in this specific drive reduces prop RPM compared to Engine RPM by a ratio of 1.5 to 1.

The counter rotating propellers were 24 and 26 inch pitch propellers. The average pitch, 25 inches, was used in the traditional prop slip equations.

First how far the boat would have gone in each 1/10 second with zero slip was calculated:

Theoretical Distance = (.1 seconds) X (Engine Revolutions/minute) X (1/Drive Gear Ratio) X (Propeller Pitch in inches) X (1 foot/12 inches) X (1 minute/60 seconds)

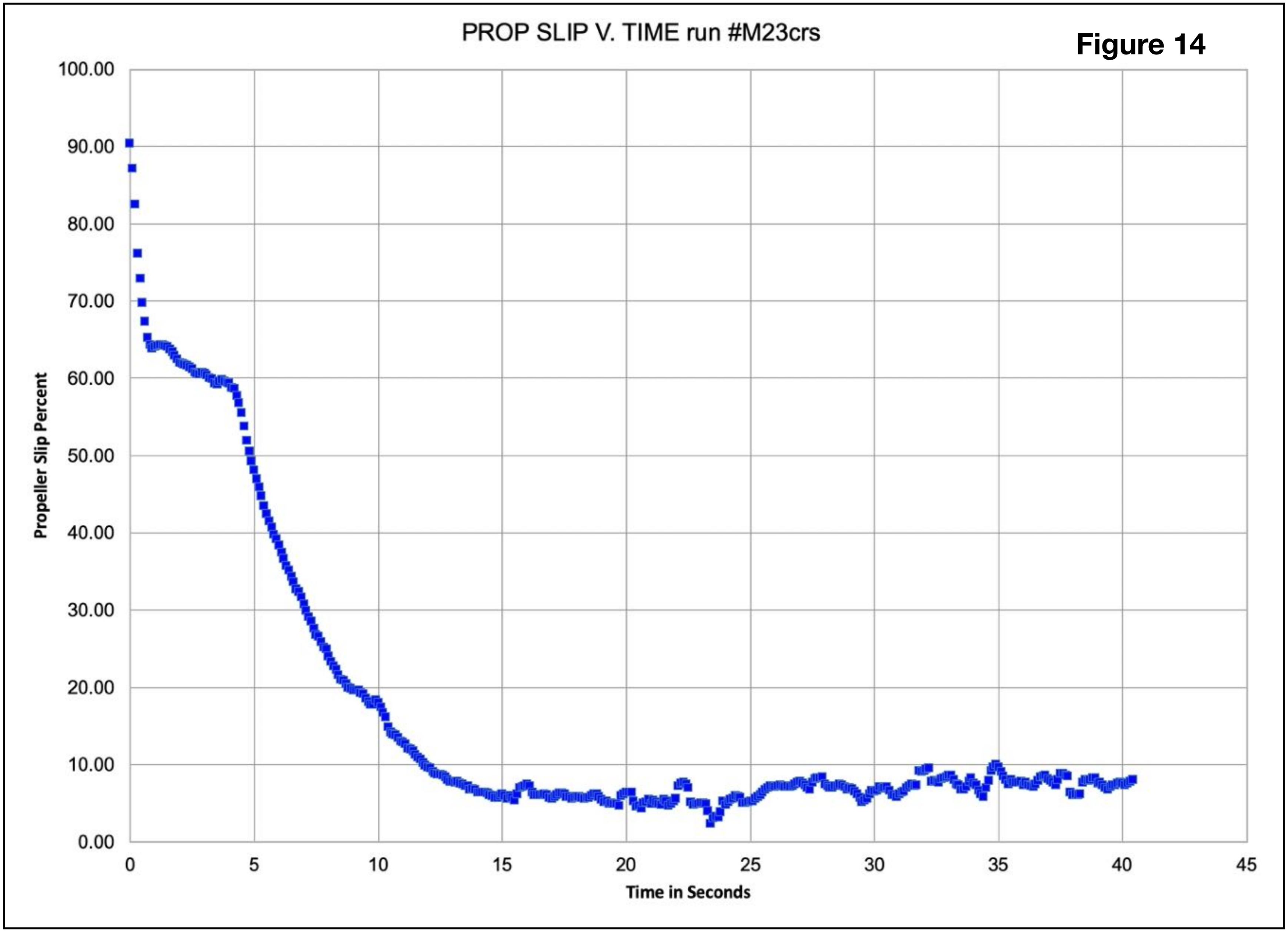
Next the actual distance the boat traveled each 1/10th of a second was calculated by multiplying the velocity at the end of that tenth of a second times 1/10 of a second (the time step).

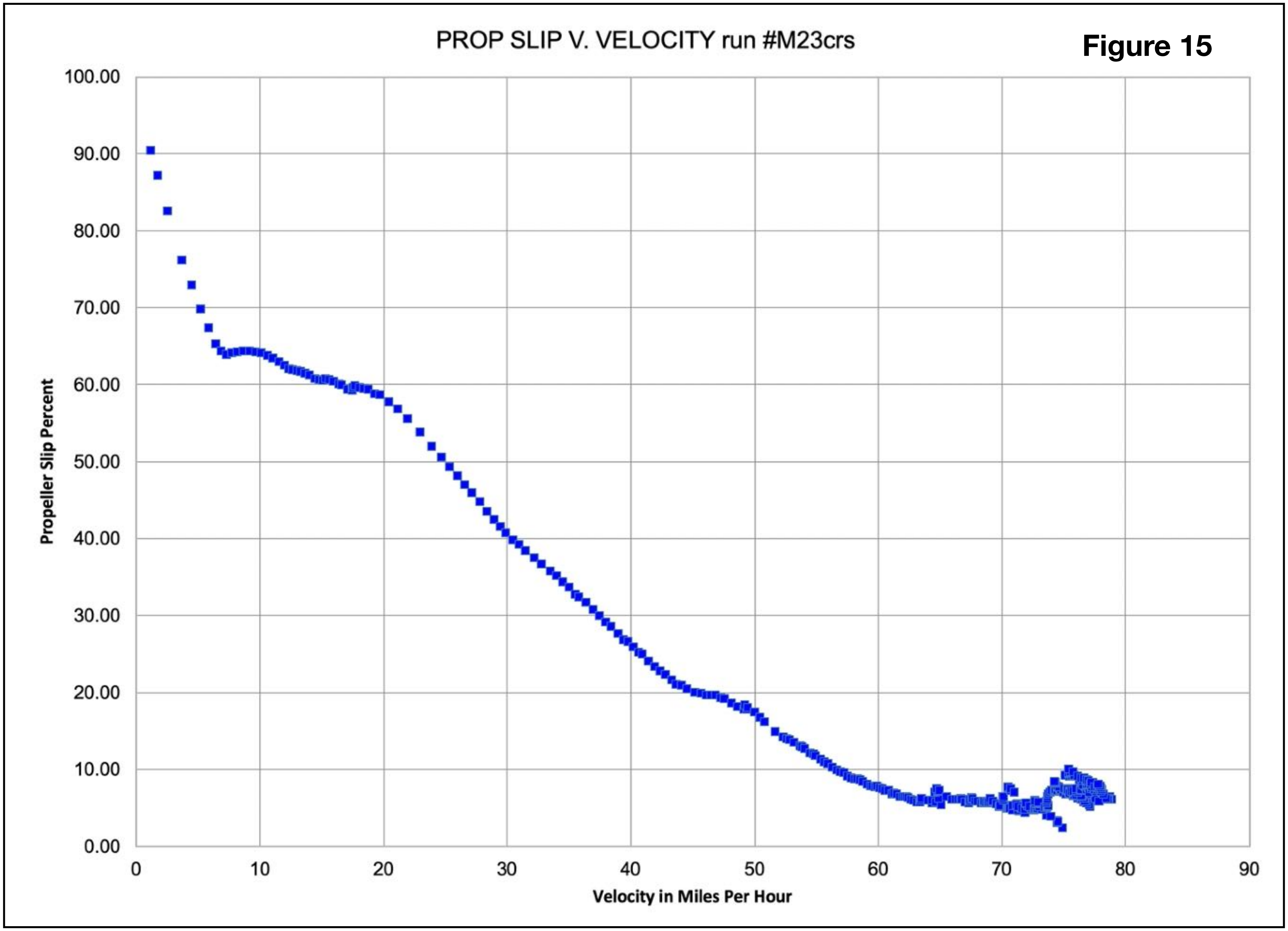
Propeller Slip = (1 - Actual Distance/Theoretical Distance) X 100 percent

During a run, initially prop slip is near 100 percent (propeller is turning but the boat is not going anywhere). As time goes on, the boat begins to plane at about 4 seconds and propeller slip begins to decrease more quickly because the boat is easier to push (it is on plane). As the boat begins to reach top speed, prop slip begins to level out at some minimum value.

Propeller Slip vs Velocity, Figure 15

The Propeller Slip vs Velocity curve plots propeller slip data every 1/10 of a second. The propeller slip vs velocity curve, **Figure 15**, looks like the propeller slip vs time curve, **Figure 14**, because velocity is increasing over the same time span that elapsed time is increasing. The inflection point at about 8 miles per hour may be attributed to the propellers beginning to surface and the inflection point at about 18 miles per hour to the boat going on plane.


PROP SLIP V. TIME run #M23crs
Figure 14
Propeller Slip Percent
100.00
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
0
5
10
15
20
25
30
35
40
45
Time in Seconds


PROP SLIP V. VELOCITY run #M23crs
Figure 15
Propeller Slip Percent
100.00
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
0
10
20
30
40
50
60
70
80
90
Velocity in Miles Per Hour

Top Velocity, Figure 16

The Lotus 123 macro defines top velocity as the single highest speed in the velocity column. It drew a line across the Velocity vs Time chart to allow visual interpretation of the speed really being the top speed or not as seen in **Figure 16**. Later the data was curve fit to more accurately define top speed as seen in **Figure 26**.

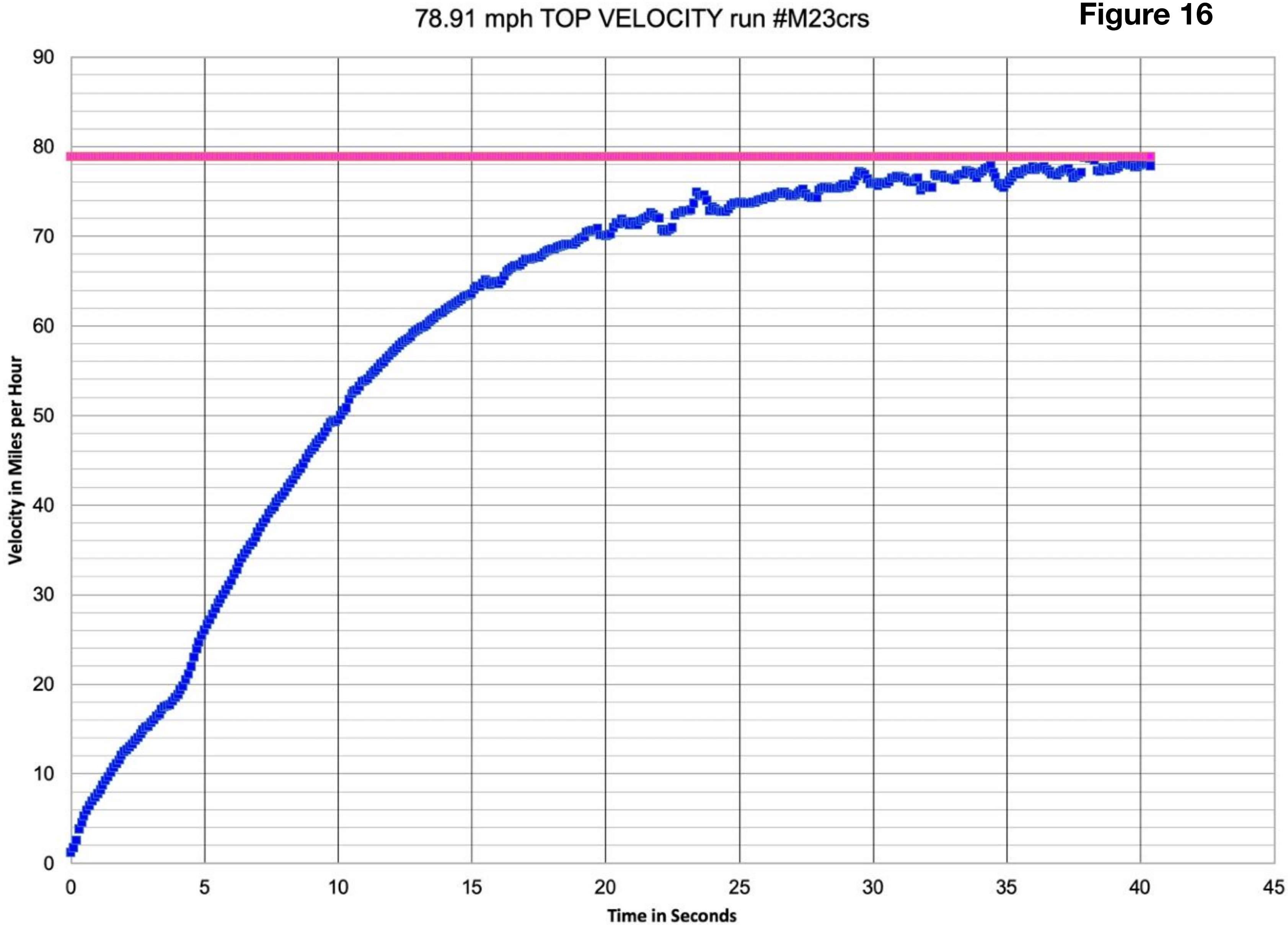
RPM Jump, Figure 17

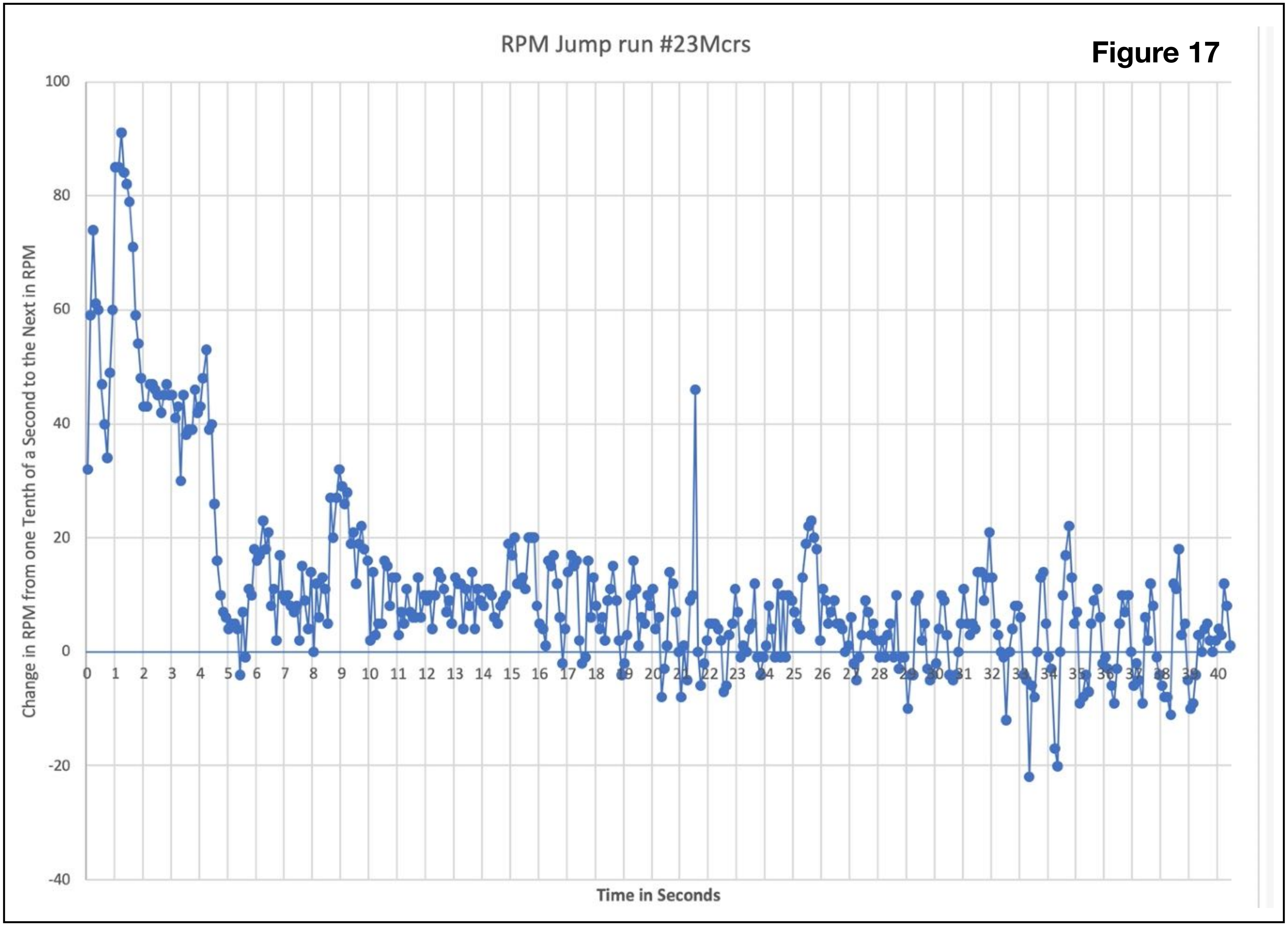
Change in RPM each tenth of a second was plotted in **Figure 17**. This curve is a precursor of the next curve where the point in time the boat took off to begin the high speed run is determined.

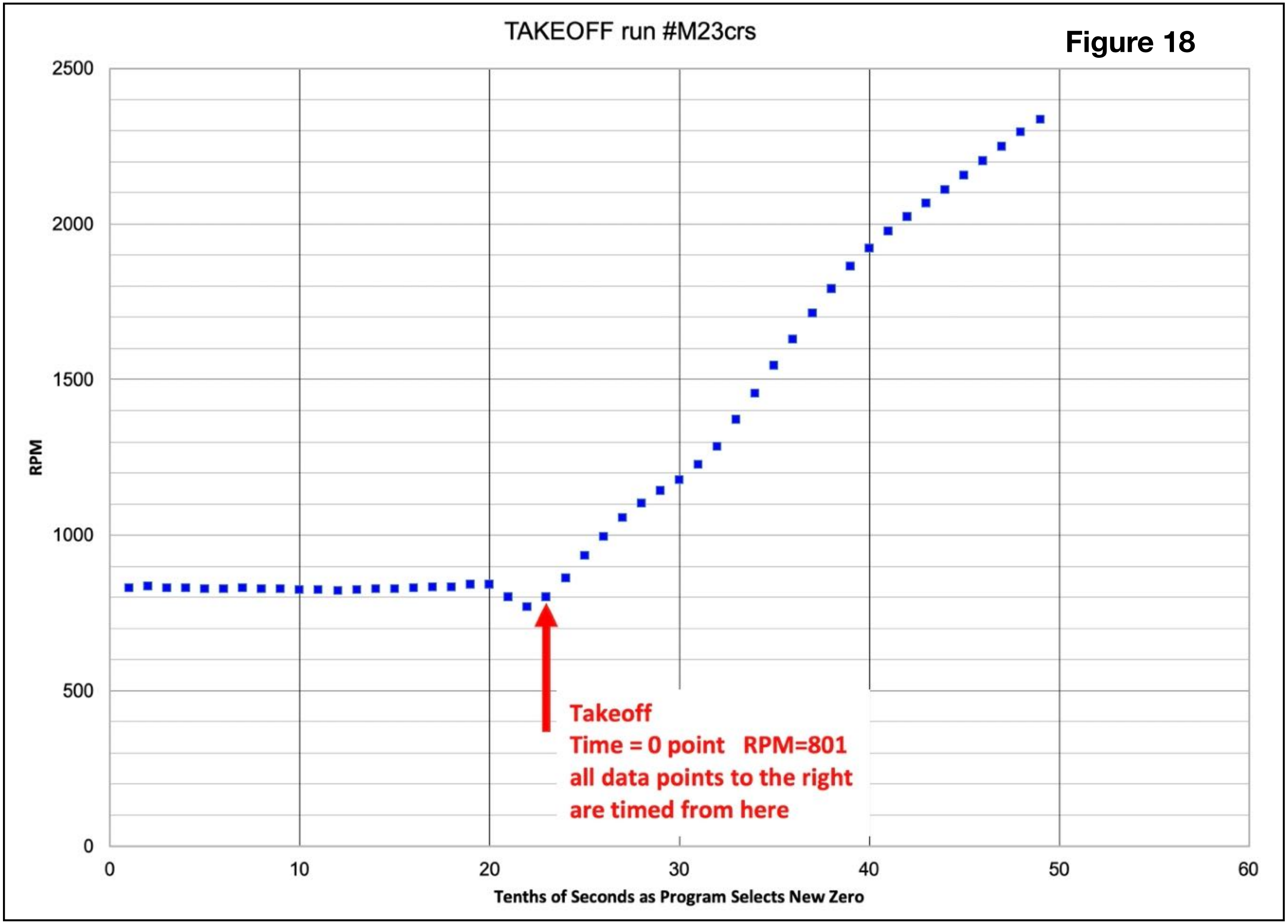
Take Off Detail, Figure 18

An algorithm stepped through engine RPM data during the performance run to detect when the throttle was thrown forward. The algorithm searched for when the RPM jumped by a specified amount or more (40 RPM was used) in a tenth of second twice in a row to prevent false positives. Time data was then re-written to use the proper take off time as zero. The macro provided **Figure 18**, a zoomed in chart of the start point selected as compared to its neighboring times and RPM values. The macro also provided the option of manually setting start time if desired. The surfacing drive with its large, counterrotating, fully submerged propellers at takeoff was particularly challenging for the macro to determine takeoff time. For example, in **Figure 18** the takeoff point could be identified as being as many as three time steps (.3 seconds) to the left (earlier).

Figure 16


78.91 mph TOP VELOCITY run #M23crs
Velocity in Miles per Hour
Time in Seconds
0
10
20
30
40
50
60
70
80
90
0
5
10
15
20
25
30
35
40
45


RPM Jump run #23Mcrs
Figure 17
Change in RPM from one Tenth of a Second to the Next in RPM
100
80
60
40
20
0
-20
-40
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
Time in Seconds


TAKEOFF run #M23crs
Figure 18
RPM
2500
2000
1500
1000
500
0
0
10
20
30
40
50
60
Tenths of Seconds as Program Selects New Zero
Takeoff
Time = 0 point RPM=801
all data points to the right
are timed from here

Blank Page

Curve Fitting Data From a Top Speed Run

Curve Fitting the Displacement Portion of the Top Speed Curve, Figure 19 & Figure 20

In 2020, a trial version of Curve Expert Pro was used to curve fit the displacement portion of the velocity curve of the example (#M23crs) used in this paper. Thanks to Curve Expert Pro for use of their software. Their watermark is visible on the charts. Below is the displacement curve previously developed on **Page 19**.

$$V_d = A_d\,(1 - e^{-b_d(t-t0_d)})$$

The 9 Point Moving Chord Acceleration Curve (**Figure 9**) revealed a discontinuity from about 3.5 to 4.5 seconds in which acceleration is rapidly increasing. The boat is transitioning from displacement mode to planing mode. After examining the individual data points, the transition zone was identified as being >3.2 seconds to < 4.7 seconds. The transition zone was eliminated from the displacement mode and planing mode for curve fitting purposes. Top Speed Run displacement mode velocity data was curve fit from takeoff to 3.2 seconds.

Curve Expert Pro was used to curve fit the displacement mode equation.

In Curve Expert Pro, the equation was written as $y=a(1-e^{-b(x-c)})$

where **a** represents A_d , **b** represents **b**, and **c** represents **T0**

Curve Expert Pro calculated displacement mode variables as shown below.

$$V_d \text{ miles per hour} = 23.81(1 - e^{-.3351(t+.1846)})$$

The curve fits quite well, see **Figure 19**. The fitted curve is just a little high for the first 3 data points and a little low for the next five data points. This discrepancy may be due to slow response of the pitot tube at takeoff and to high loads surfacing counterrotating propeller drives place on the engine under hard acceleration when propellers are fully submerged at takeoff as seen in **Figure 18**.

Figure 20 is an enlarged view of the graph portion of **Figure 19**.

Figure 19: Curve Fit Displacement Curve

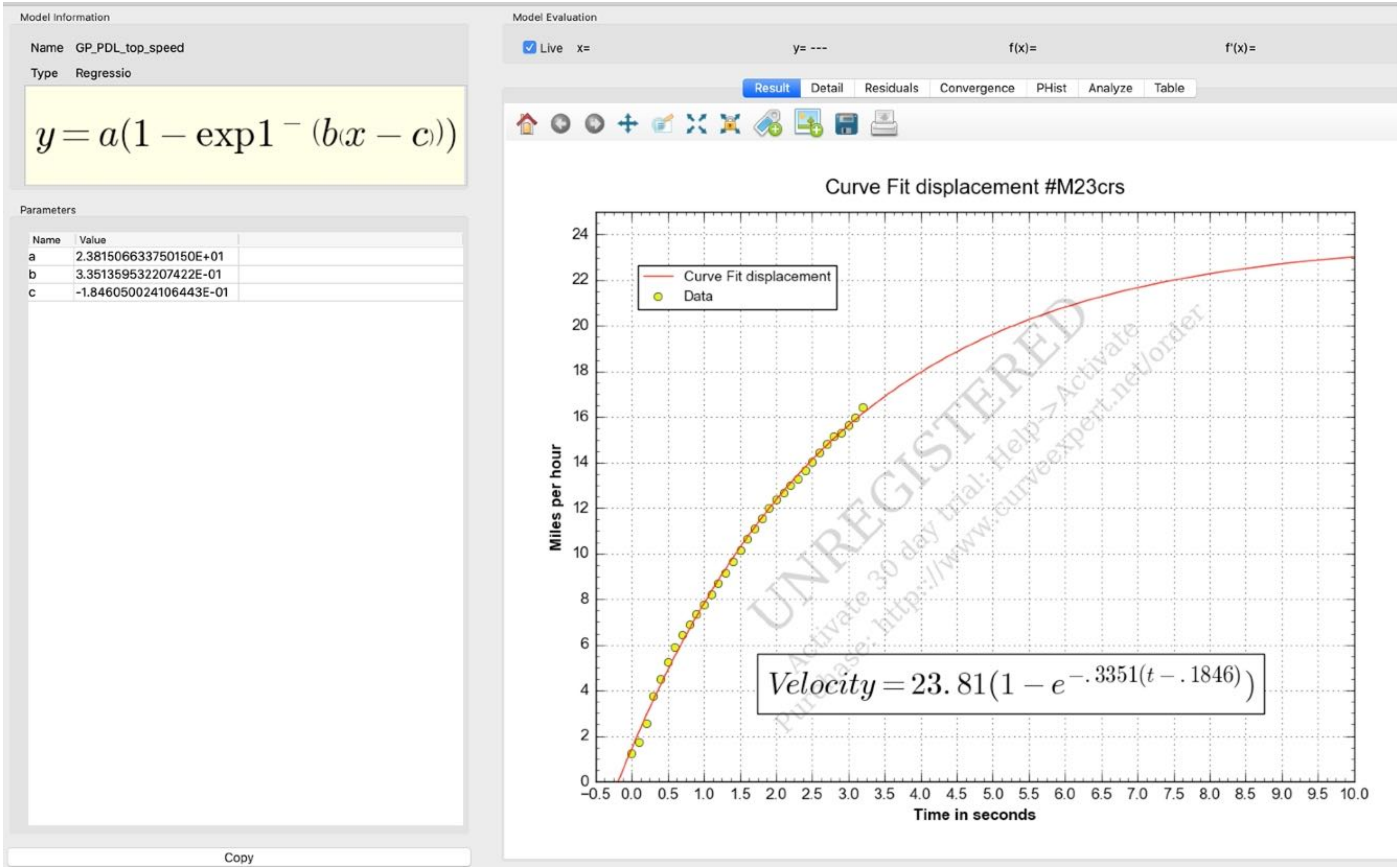
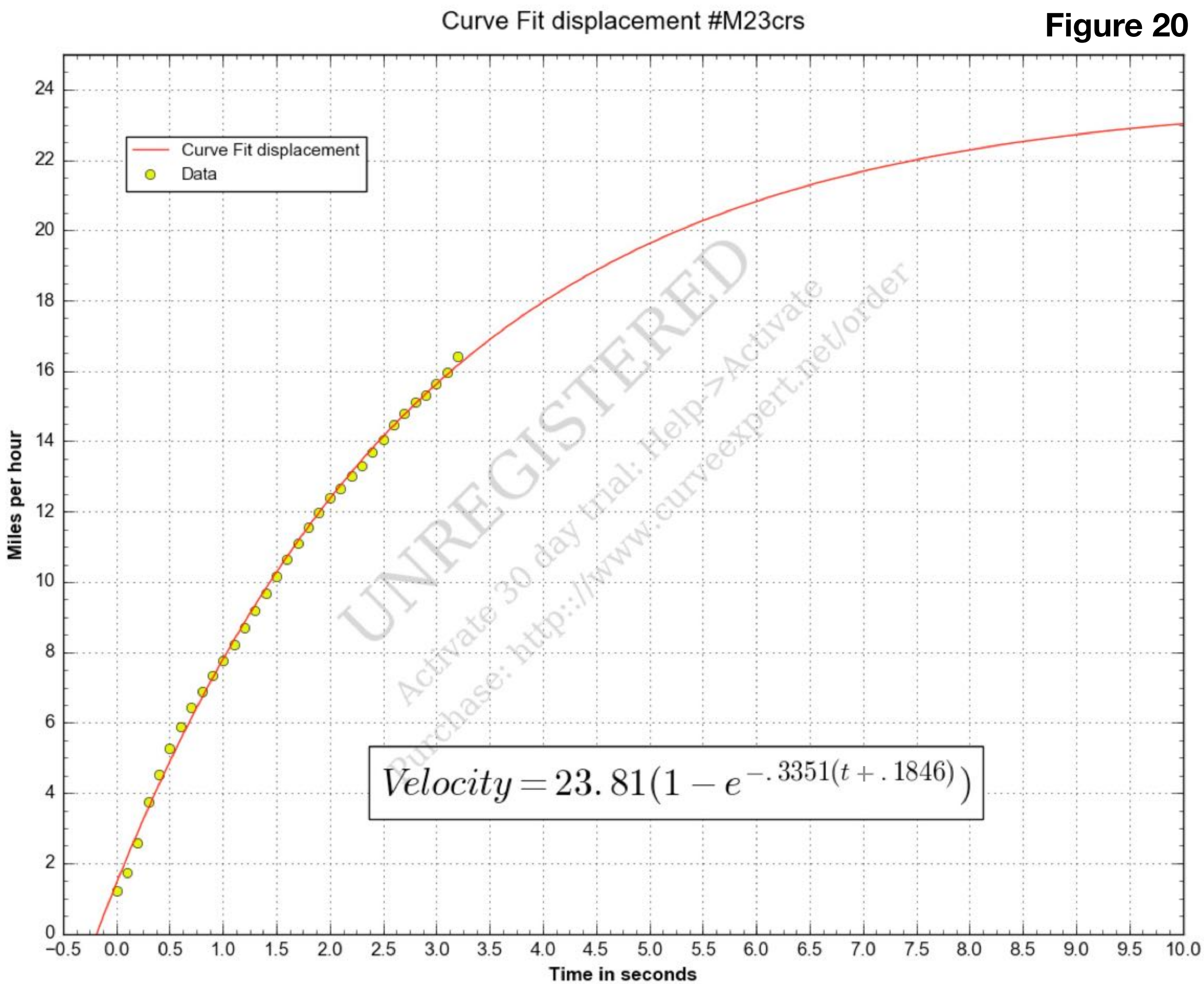


Figure 20


Curve Fit displacement #M23crs
Curve Fit displacement
Data
Miles per hour
Time in seconds
0
2
4
6
8
10
12
14
16
18
20
22
24
−0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5 9.0 9.5 10.0
UNREGISTERED
Activate 30 day trial: Help->Activate
Purchase: http:://www.curveexpert.net/order
Velocity = 23.81(1 − e^{−.3351(t + .1846)})

Displacement Mode Residuals, Figure 21

Curve Expert Pro furnished a residuals plot of differences between actual velocity values and the curve fit velocity values each tenth of a second in **Figure 21**. The first 3/4 of the residuals looks sinusoidal (like a sine wave) vs being totally random. Some of the sinusoidal response is likely due to conditions mentioned earlier (pitot tube response and large fully submerged counterrotating propellers being challenging to spin at takeoff).

Curve Fitting Coefficients

Velocity of this boat under hard acceleration during displacement phase can be accurately represented by just three coefficients (**A**, **b**, and **t0**).

Historical efforts to model boat performance in a top speed or coast down run have typically tried to curve fit velocity v. time data with polynomials. For example:

$$\textbf{Velocity} = a + bx + cx^2 + dx^3 + ex^4$$

Those trying to model boat top speed runs or coast down data[9] are able to get a reasonable fit from polynomial equations. However the resulting polynomial coefficients have no meaning or basis in physics vs the coefficients in our model. Additionally, the model revealed in this paper is able to model boats in top speed runs with just three coefficients vs five when modeling with polynomials.

The Equation & MathText, Figure 22

The equation shown in the upper left corner of **Figure 19** and **Figure 25** and highlighted in **Figure 22** looks different from the curve being curve fit due to how Curve Expert Pro writes equations.

Curve Expert Professional uses at least three methods of representing equations.

1. Somewhat normally with exp1 to represent the scientific constant e and ** to indicate an entity is raised to a power.
2. Python computer language.
3. MathText is a way of writing mathematical expressions used in MatPlotLib (a Python library) and some other scientific programs. MathText was used to print equations directly on Figures as an annotation.

---

[9] Toward Reconstructing Minimum Speeds in Recreational Boating Accidents. Bruce W. Main. Miller Engineering, Inc. Society of Automotive Engineers (SAE) International Congress and Exhibition. Detroit Michigan. February 27 - March 2. 1995. SAE Technical Paper Series. 950732.

Figure 21. Residuals of Curve Fit of Displacement Mode of Curve #M23crs

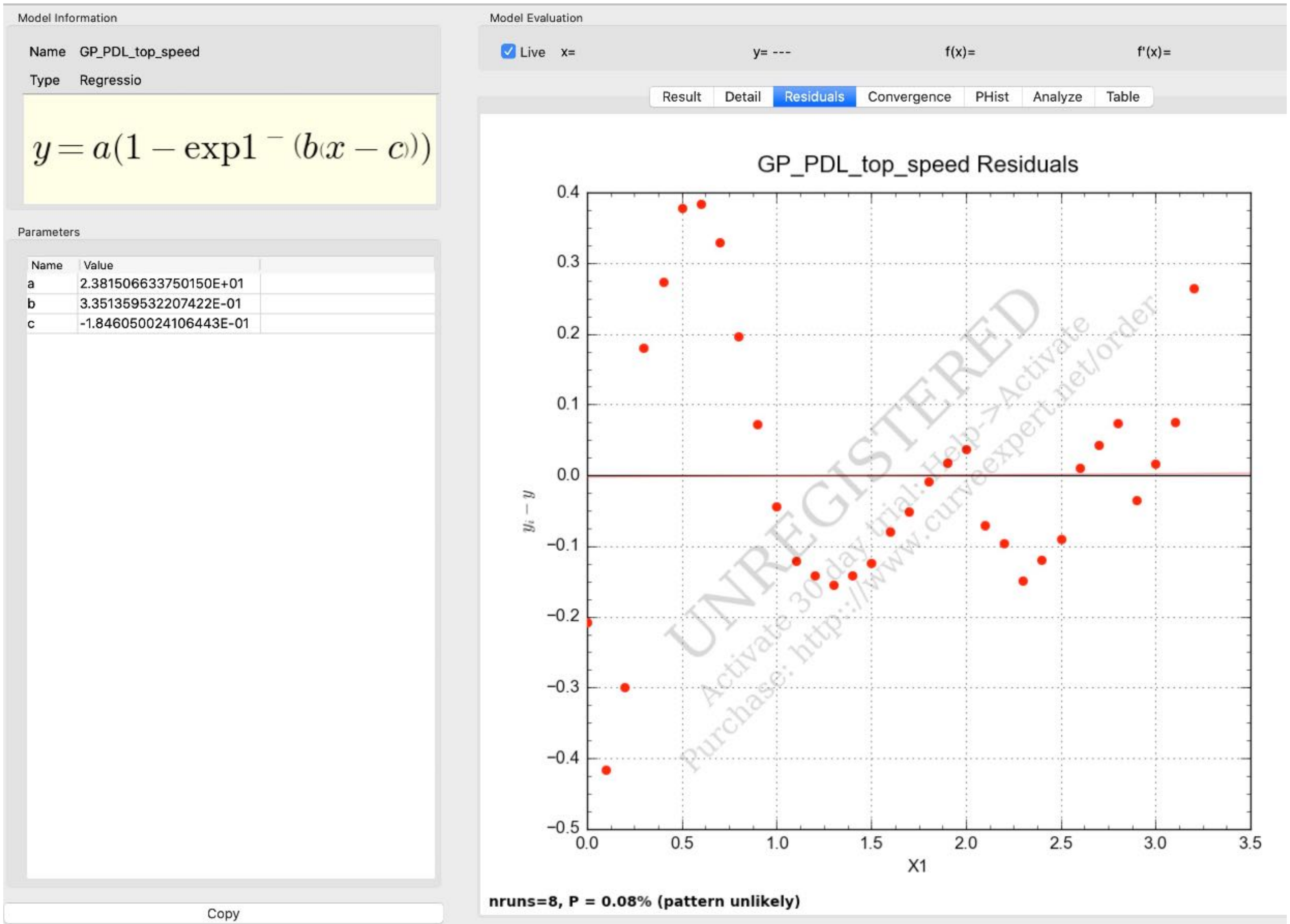


Figure 22: Equations

Name GP_PDL_top_speed

Type Regressio

$$y = a(1 - \exp 1^{-}(b(x - c)))$$

Parameters

Name	Value	
a	2.381506633750150E+01	
b	3.351359532207422E-01	
c	-1.846050024106443E-01	

Equation in Upper Left Corner of Figure 19 and Figure 21

$$Velocity = 23.81(1 - e^{-.3351(t + .1846)})$$

The Resulting Equation as Seen in Figure 19 and Figure 20

Entering the Equation, Figure 23

The yellow box in **Figure 23** shows how the equation was entered in Curve Expert Pro.

In order to help the equation quickly hone in on the correct coefficients to best curve fit the data, the system instructs users to guess a starting value for the coefficients. Guesses do not have to be very close to the actual final value, they just give the program a place to start. In the instance of Run #M23crs boat velocity, the curve fitting part of the program refused to run without the user first entering a guess. The program kept returning “Catastrophical Failure” until guesses were entered.

Strength of Curve Fit of the Displacement Mode of the Velocity Curve, Figure 24

Curve Expert Pro provides some statistics on how accurately the curve fits the data. In this instance it reports the Correlation Coefficient (r) equals .999116. A perfect fit would have the value of 1. This curve is a very good fit.

The Coefficient of Determination, or r^2 equals .998231900309. Again, this is a very good fit.

Curve Expert Pro also provides a range of the coefficients in which there is a 95 percent confidence range for the actual value of each coefficient.

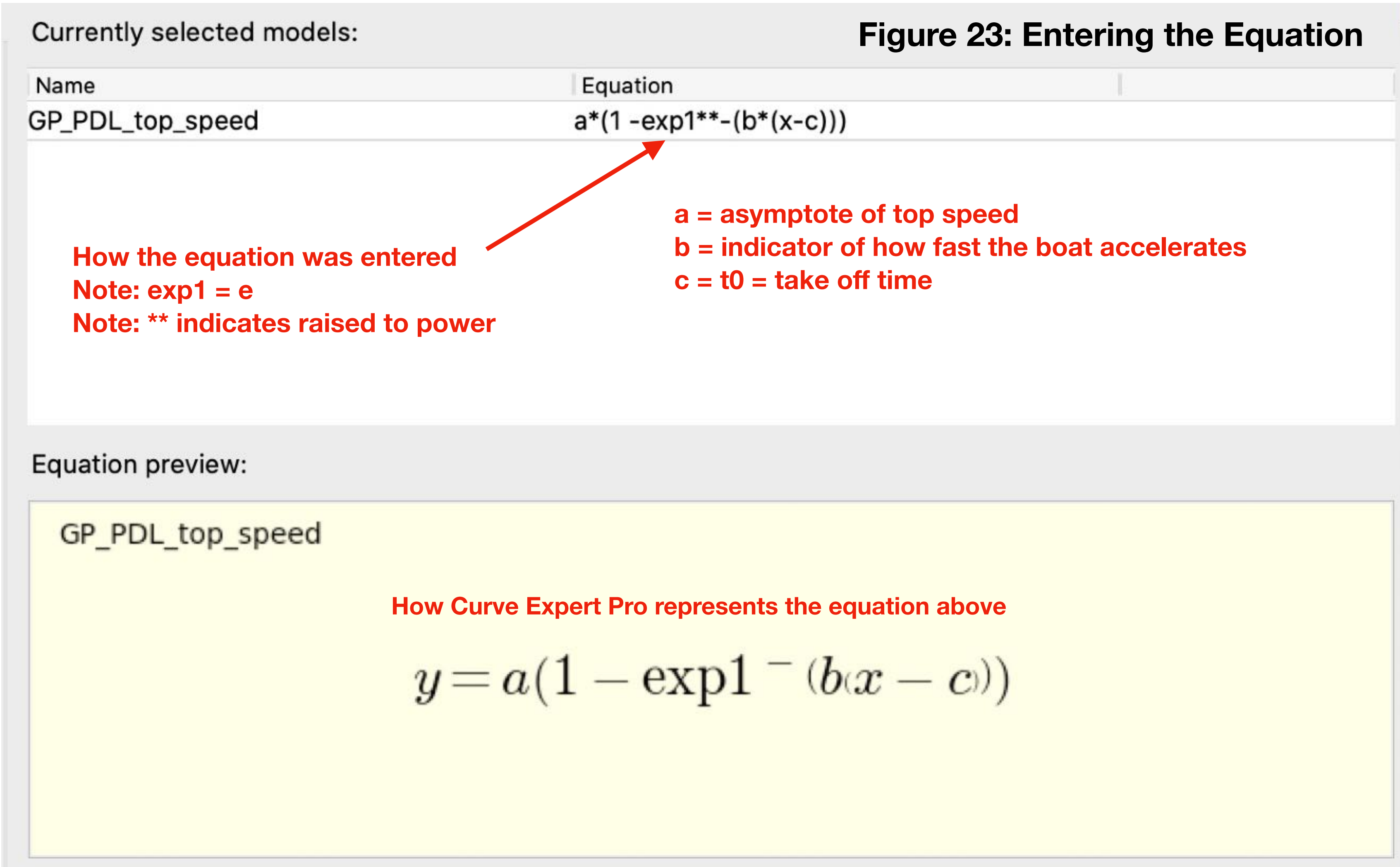
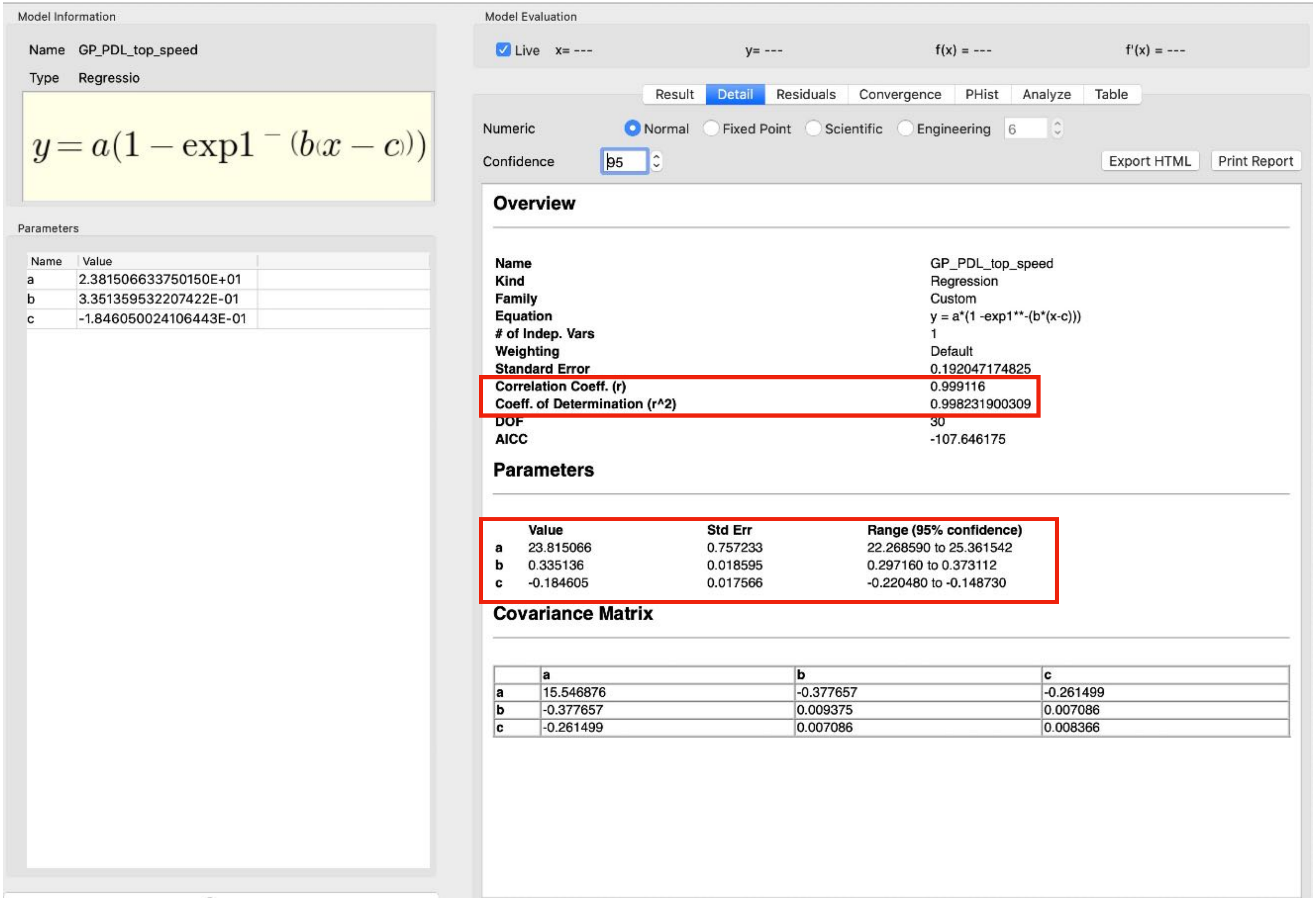


Figure 23: Entering the Equation

Figure 24: Strength of Curve Fit of the Displacement Mode Run#M23crs



Curve Fitting the Planing Portion of the Top Speed Curve, Figure 25 and Figure 26

Similar to the displacement portion curve, a trial version of Curve Expert Pro was used to curve fit the planing portion of the velocity curve. Below is the planing curve previously developed on **Page 19**.

$$V_p = A_p\,(1 - e^{-b_p(t-t0_p)})$$

As discussed on **Page 59**, the boat is transitioning from displacement mode to planing mode from >3.2 seconds to < 4.7 seconds. Thus the curve fit the planing mode data for time greater than or equal to 4.7 seconds.

Curve Expert Pro was used to curve fit **a** as A_d , **b** as **b**, and **c** as **T0** similar to curve fitting the displacement mode on **Page 59**, resulting in:

$$V_d \text{ miles per hour } = 78.12(1 - e^{-.1283(t-1.950)})$$

The curve above fits quite well, see **Figure 25**. However the velocity data gets a little bouncy as the drive begins to approach top speed. The bouncing observed in the velocity data is likely partially due to the operator trimming the boat for maximum speed.

Figure 26 is an enlarged view of the graph portion of **Figure 25**.

Figure 25: Curve Fit Planing Curve

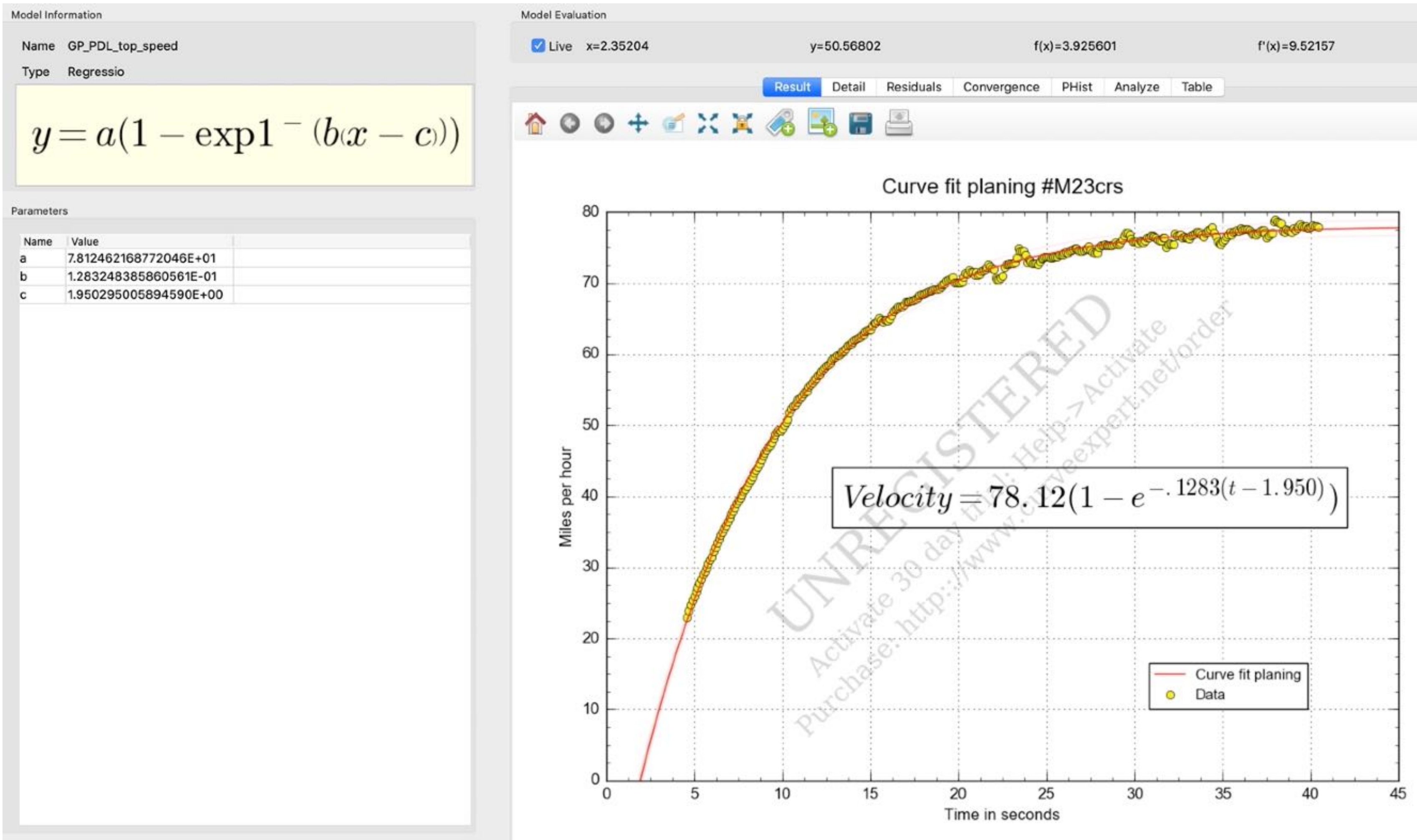
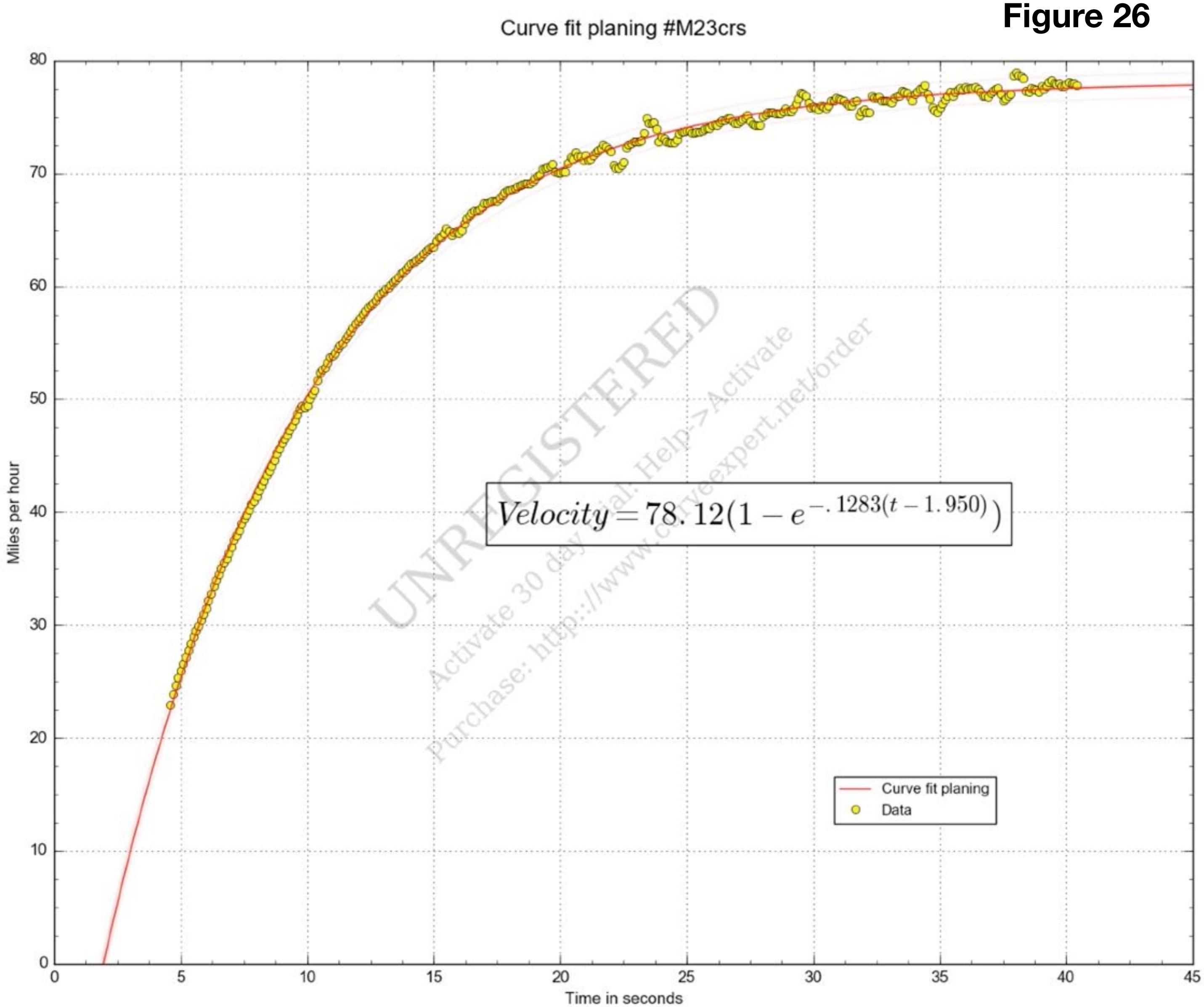


Figure 26


Curve fit planing #M23crs
Miles per hour
Time in seconds
$Velocity = 78.12(1 - e^{-.1283(t - 1.950)})$
Curve fit planing
Data
UNREGISTERED
Activate 30 day trial: Help->Activate
Purchase: http://www.curveexpert.net/order
0
10
20
30
40
50
60
70
80
0
5
10
15
20
25
30
35
40
45

Planing Mode Residuals, Figure 27

The first one-third of Residuals of the Planing Curve Mode of the Velocity Curve (**Figure 27**) looks sinusoidal (sine wave) vs being totally random. The remainder of residuals look reasonably random. Above about 20 seconds there are some larger residuals in the region the boat operator is adjusting trim as they hunt for maximum speed.

Strength of the Planing Mode Curve Fit, Figure 28

Curve Expert Pro provides some statistics on how accurately the curve fits the data. For the planing portion of the curve it reports the Correlation Coefficient (r) equals .999269. This curve is a very good fit.

The Coefficient of Determination, or r^2 equals .9985380366. A very good fit.

Curve Expert Pro also provides a range of the coefficients in which there is a 95 percent confidence range for the actual value of each coefficient.

Figure 27 Residuals of Curve Fit of Planing Mode of Velocity Curve Run #M23crs

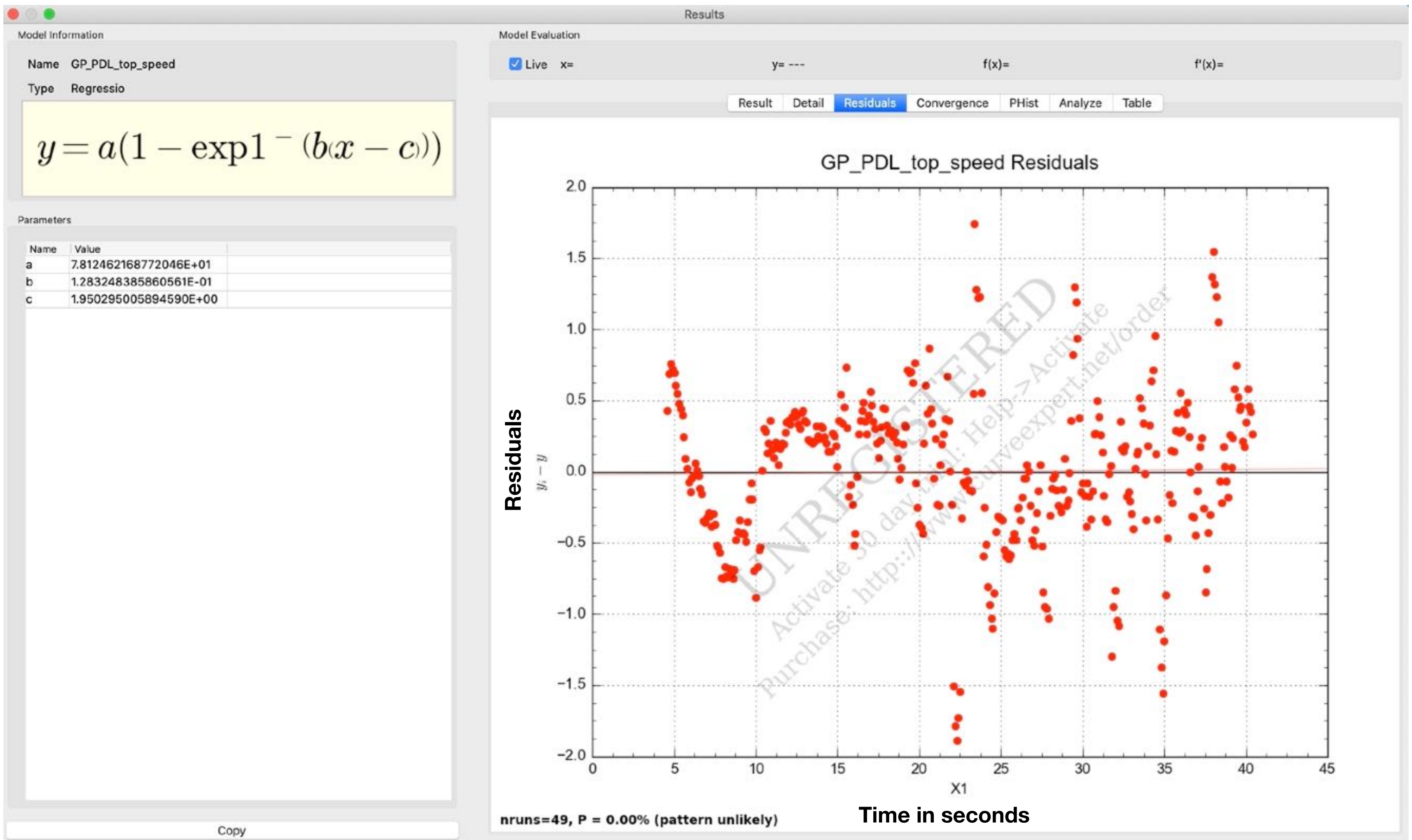
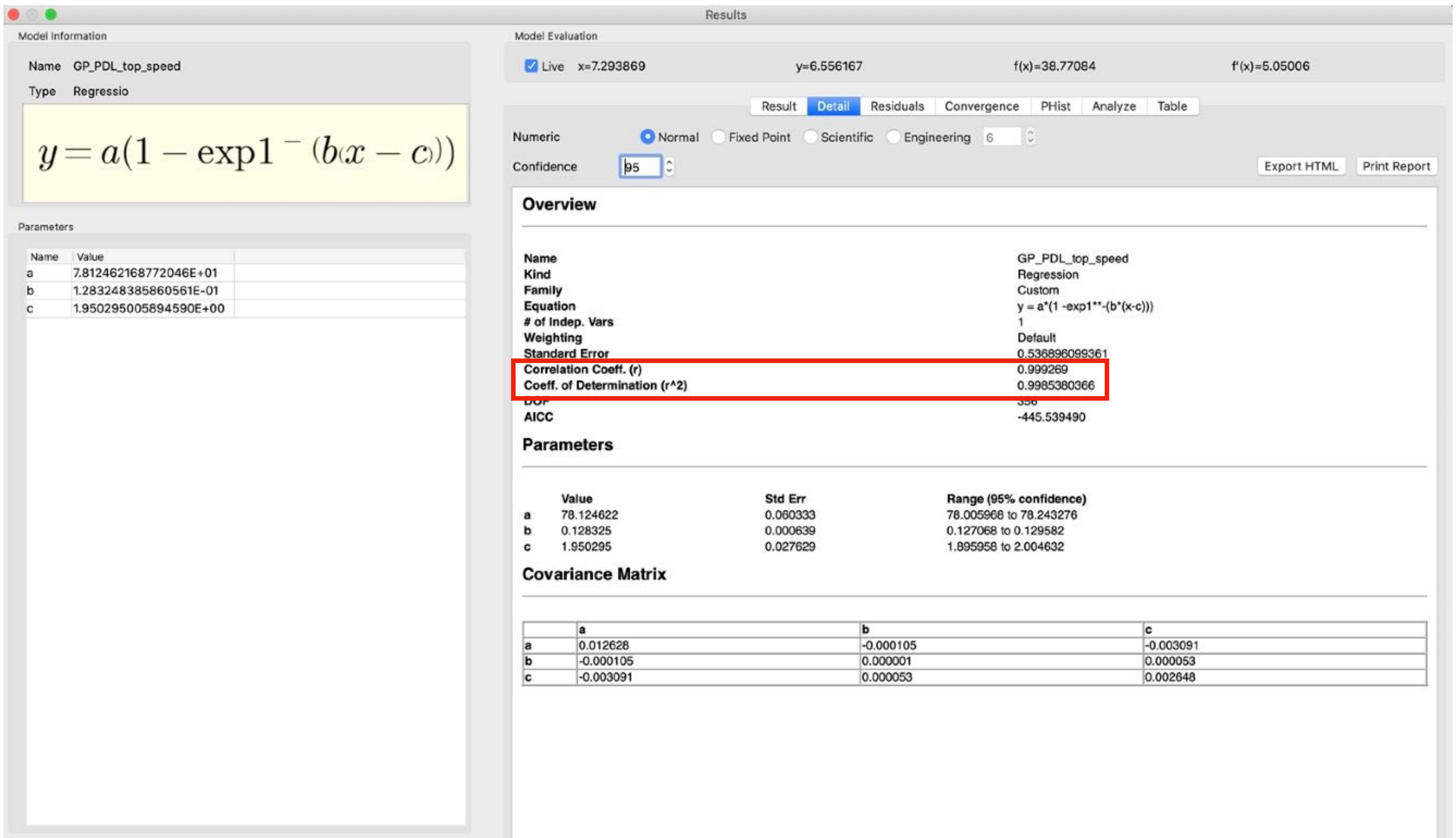


Figure 28: Strength of Curve Fit of Planing Mode Velocity Run #M23crs



Calculating Propeller Thrust Using Derivative Method: Run #M23crs Displacement Mode

As per our earlier example, on **Pages 27 & 28:**

Acceleration at takeoff in **Figure 3** can now be estimated as **A** times **b**.

A = 23.81 miles per hour

b = .3351 / seconds

acceleration = $A_d \times b_d$ = (23.81 miles/hr) × (.3351 / seconds) × (5280 ft/mile) × (1 hour / 3600 seconds)

acceleration at takeoff = 11.70 ft/sec^2

Boat weight was not available so W was used for boat weight in the equation below.

Boat Mass = W pounds / 32.2 = .0311 x W slugs

Thrust at takeoff = Mass × acceleration

Thrust at takeoff = .0311 x W slugs × 11.70 ft/sec^2

Thrust at takeoff = .3639 X W pounds or roughly 1/3 X boat weight in pounds

Calculating Propeller Thrust Using Derivative Method: Run #M23crs Planing Mode

As per our displacement example, on **Page 74:**

Acceleration at takeoff in **Figure 3** can now be estimated as **A** times **b**.

A = 78.12 miles per hour

b = .1283 / seconds

acceleration = $A_d \times b_d$ = (78.12 miles/hr) × (.1283 / seconds) × (5280 ft/mile) × (1 hour / 3600 seconds)

acceleration at takeoff = 14.70 ft/sec^2

Boat weight was not available so W was used to represent boat weight in the equation below.

Boat Mass = W pounds / 32.2 = .0311 x W slugs

Thrust at takeoff = Mass × acceleration

Thrust at theoretical takeoff point (where the planing curve intercepts the X axis) = .0311 x W slugs × 14.70 ft/sec^2

Thrust at takeoff = .4572 X W pounds or roughly 1/2 X boat weight in pounds

Note - takeoff point for the planing run is not a real point. It is where the boat would have taken off if it took off in planing mode.

Transition Zone Figure 29 and Figure 30

Some of earlier charts of the #M23crs run data left out the transition zone data in which the boat transitions from displacement to planing mode.

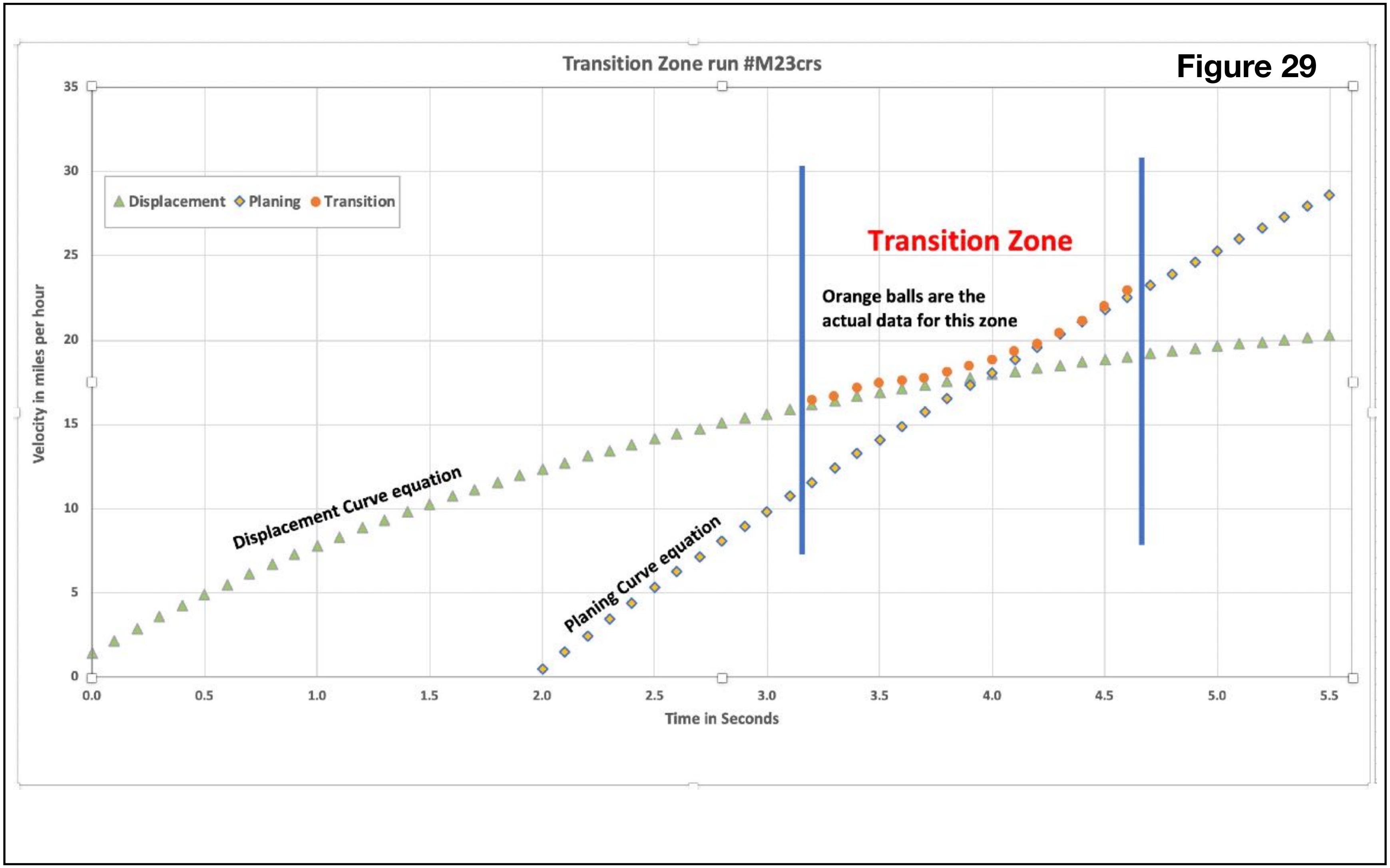
Microsoft Excel was used to calculate boat velocity each tenth of a second in the displacement and planing modes using the equations derived in the curve fit section of this report. Those velocities (the velocities derived from using the curve fit equations, not the actual velocities) are shown in **Figure 29** for the displacement and planing portions of the velocity curve.

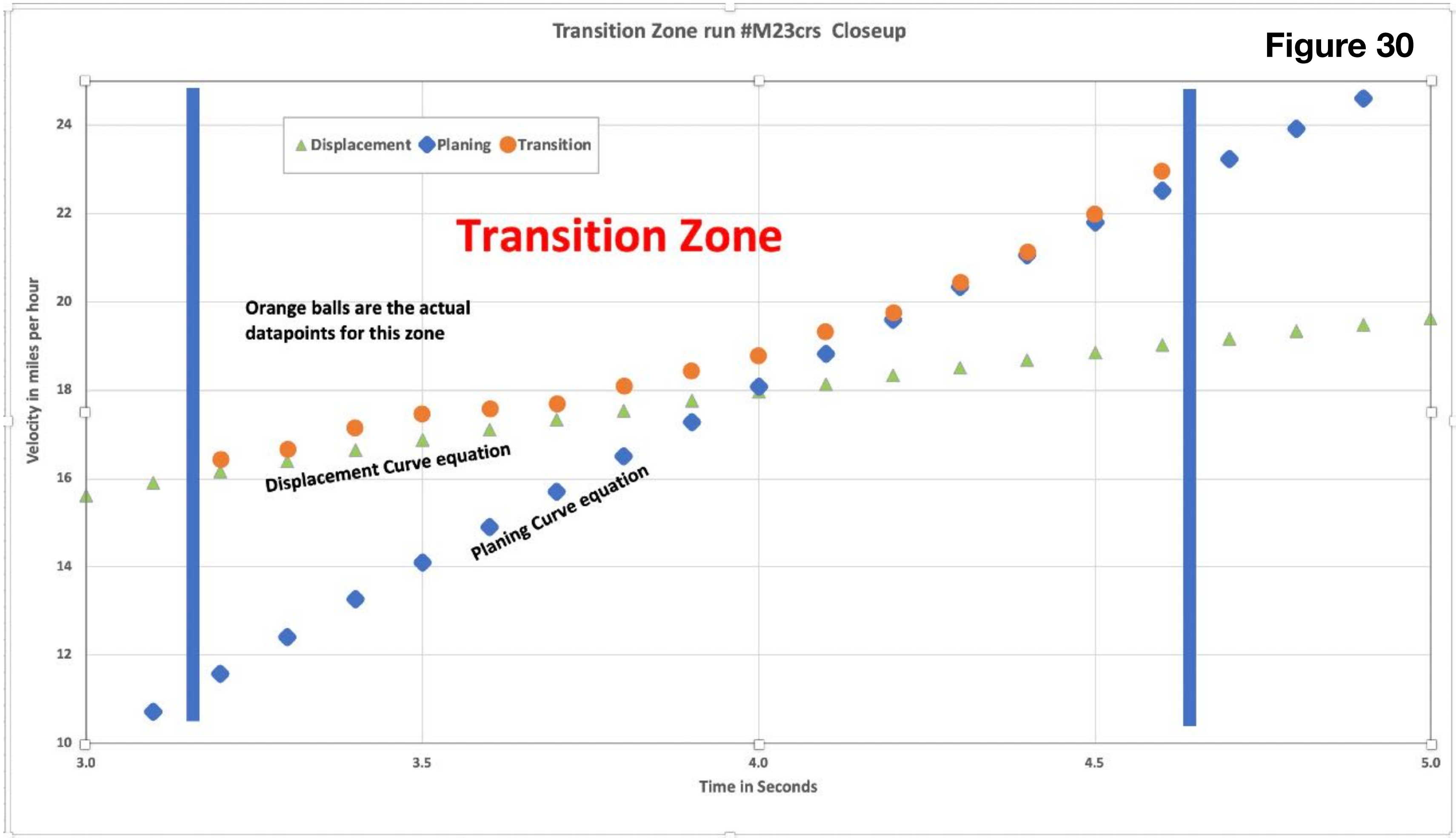
Actual boat velocities in the transition zone were plotted as orange dots/disks.

The transition portion of **Figure 29** is enlarged in **Figure 30**.

You can see and compare velocities calculated from the displacement and planing curve equations in the region of 4 seconds (the transition zone).

In **Figure 30** boat velocity begins to increase to a little faster than the velocity calculated by the displacement equation as the Transition Zone begins. The orange disks representing actual boat velocity lift up off the displacement curve and transition to the planing curve as it comes by. The displacement and planing curve equations were derived from data from this run. It is remarkable to see how closely the actual velocity curve matches the curves in the granular data and transitions between them in the Transition Zone. We have not seen the transition zone represented so strikingly using actual data before.


Transition Zone run #M23crs
Figure 29
Displacement
Planing
Transition
Transition Zone
Orange balls are the
actual data for this zone
Displacement Curve equation
Planing Curve equation
Velocity in miles per hour
Time in Seconds
35
30
25
20
15
10
5
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5



Transition Zone run #M23crs Closeup
Figure 30
Displacement
Planing
Transition
Transition Zone
Orange balls are the actual datapoints for this zone
Displacement Curve equation
Planing Curve equation
Velocity in miles per hour
24
22
20
18
16
14
12
10
3.0
3.5
4.0
4.5
5.0
Time in Seconds

Blank Page

Virtual Boat Races

Multiple Boat Top Speed Runs on the Same Chart

RPM and Velocity data can be shown for two or more boat top speed runs on the same chart. If the charts both start at time = 0 direct comparisons can be made between the runs.

As mentioned earlier, these runs can happen the same day or be years apart. They can be the same vessel with some changes or improvements running against itself, all the way up to being two vastly different vessels.

Our RPM & Velocity of Two Boats Chart (**Figure 31**) compares top speed runs between two very different boats. The Hrun boat top speed run is a performance boat with a counter rotating surfacing drive AND a two speed transmission. The Brun boat top speed run is a ski boat.

Note - in **Figure 31** Velocity curves read to the left axis while RPM curves read to the right axis.

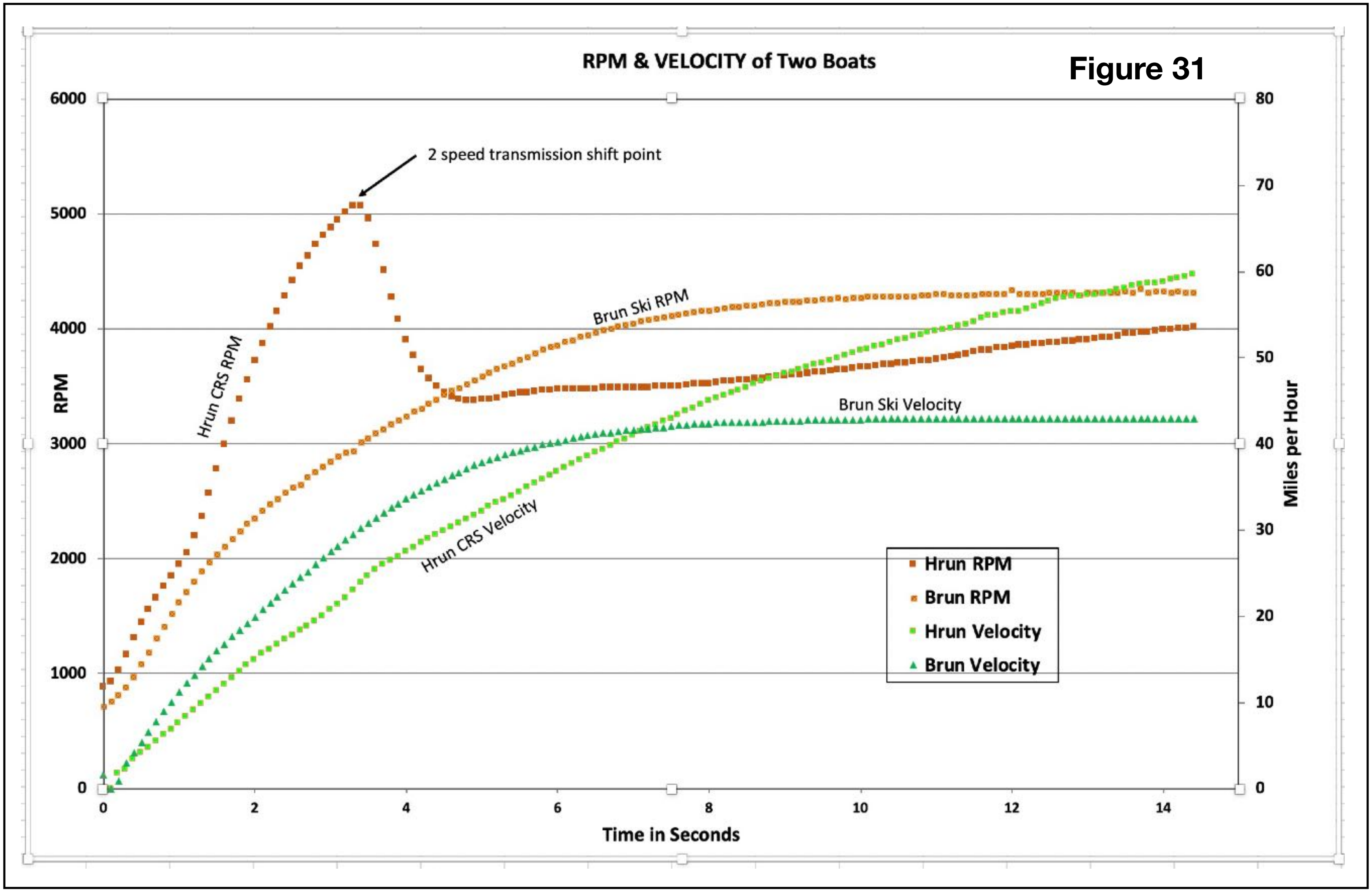
Velocity of each boat is shown in a green color. You can see the Brun ski boat quickly reaches top speed while the Hrun contra rotating surfacing propeller drive boat is still a long ways from winding up to top speed at 14 seconds into the run.

It is interesting to see the effect of a two speed transmission on these curves. The Hrun CRS boat quickly revs up to 5000 RPM and was manually shifted about 3 seconds into the run. The two speed transmission was very effective at getting the two large fully submerged counter rotating propellers spinning and quickly getting the boat up enough the propellers were surfacing which allowed them to speed up much faster.

Comparing the RPM curve in **Figure 31** (with two speed transmission) vs a similar drive in **Figure 8** (no two speed transmission) the RPM in **Figure 8** bends over at about 4 seconds as the engine struggles at winding up large propellers with its fixed gear ratio. The unit without the two speed transmission was running a gear ratio attempting to balance the requirements of taking off quickly and top speed. The drive in **Figure 31** has a two speed transmission making it more effective on both ends (take off and top speed).

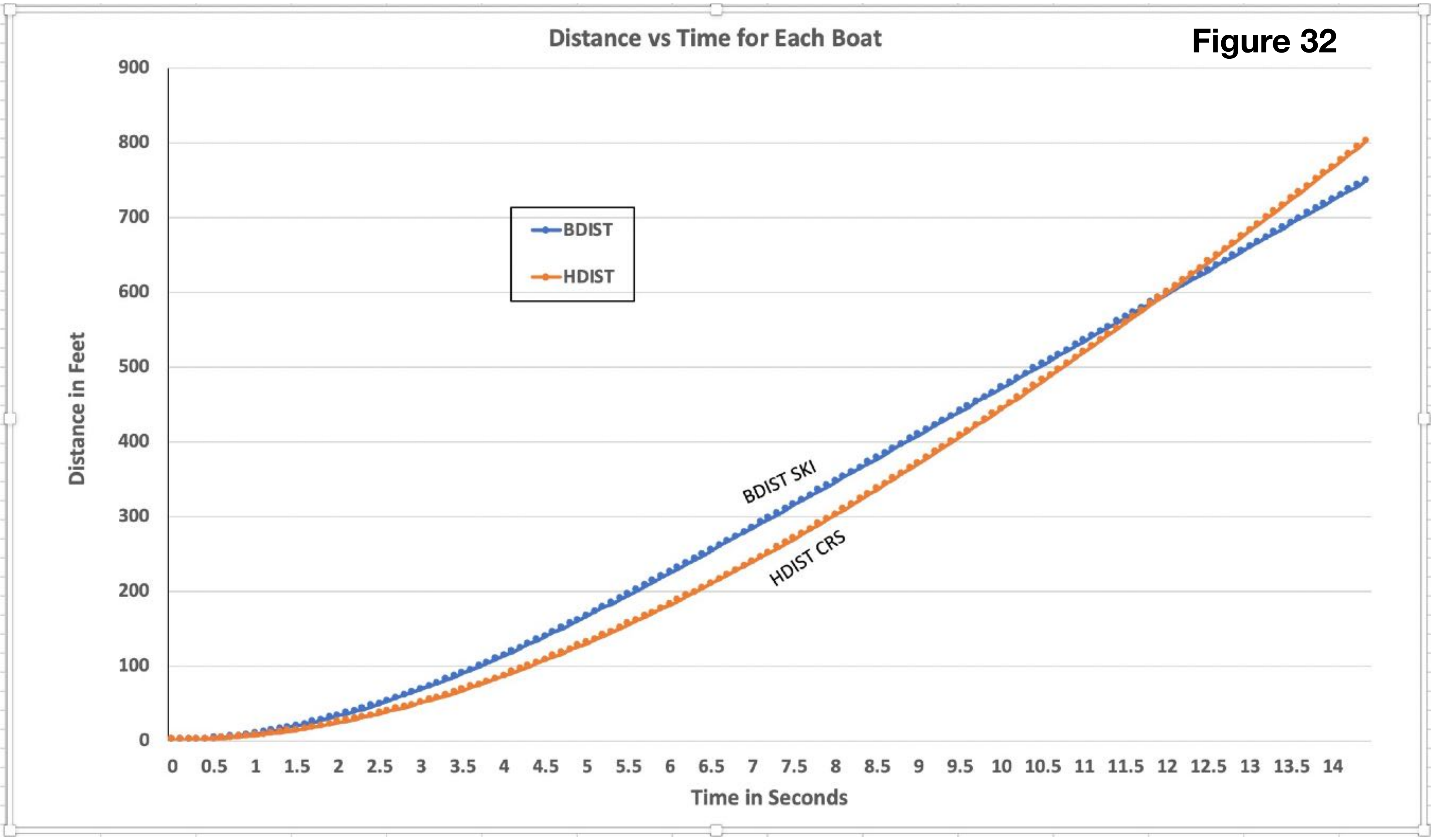
By about 7 seconds the Hrun CRS boat velocity overtook the Brun ski boat velocity. From then on, it was just a matter of time till it caught up with the ski boat and left it in the dust.

Similarly, by about 7 seconds the ski boat has reached maximum RPM while the CRS boat will take several times that long to reach maximum RPM.


RPM & VELOCITY of Two Boats
Figure 31
2 speed transmission shift point
Hrun CRS RPM
Brun Ski RPM
Brun Ski Velocity
Hrun CRS Velocity
Hrun RPM
Brun RPM
Hrun Velocity
Brun Velocity
RPM
Miles per Hour
Time in Seconds
6000
5000
4000
3000
2000
1000
0
80
70
60
50
40
30
20
10
0
0
2
4
6
8
10
12
14

Comparing Distance Traveled Between Two Top Speed Runs

Figure 32 charts distance vs time for each boat. You can see the ski boat is ahead from about 1 second to 12 seconds, then the CRS drive powered boat overtakes it. More details are available in **Figure 33**.



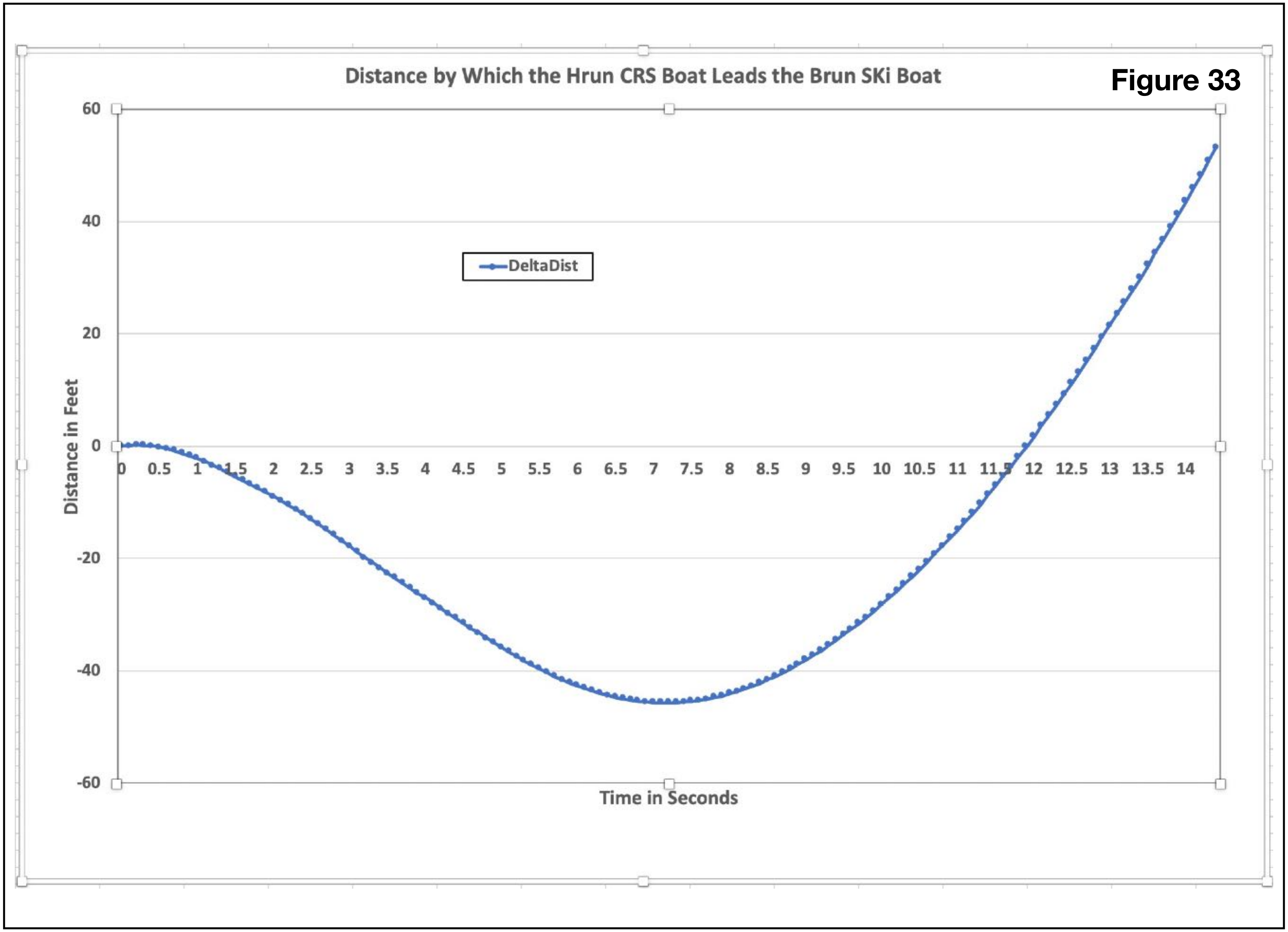
Basics of Virtual Boat Races

Having time and velocity for each each tenth of a second allows calculation of distance covered each tenth of a second. (See **Appendix A**). Distance covered each tenth of a second to the current time can be summed to obtain total distance traveled up to that time. **Figure 33** shows one way of presenting Virtual Boat Races from portable data logger data.

Figure 33 shows how far out in front the Hrun CRS powered boat would be of the Brun ski boat if they both took off at the same time in an actual race under the same conditions the boats were tested in (boat setup, weather, waves height, barometric pressure, same people on board, same fuel levels, etc.)

In **Figure 33** you can see the boats run about even for the first half of a second, then the ski boat begins to take the lead. Just like in a track and field meet, at about 7 seconds, the CRS powered boat begins to reel in the ski boat and pulls up even with it at about the 12 second mark (about 580 feet downrange from take off). After that, the CRS powered boat runs away with the race leaving the ski boat in the dust.

These virtual races can be used for many purposes including comparing the same boat with itself after a modification or improvement to the boat, marine drive, or changes in loading of the boat. Performance differences achieved by strapping on a higher horsepower motor including any weight differences due to changing the motor can be quickly understood when portrayed in this manner.


Distance by Which the Hrun CRS Boat Leads the Brun SKi Boat
Figure 33
DeltaDist
Distance in Feet
60
40
20
0
-20
-40
-60
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10 10.5 11 11.5 12 12.5 13 13.5 14
Time in Seconds

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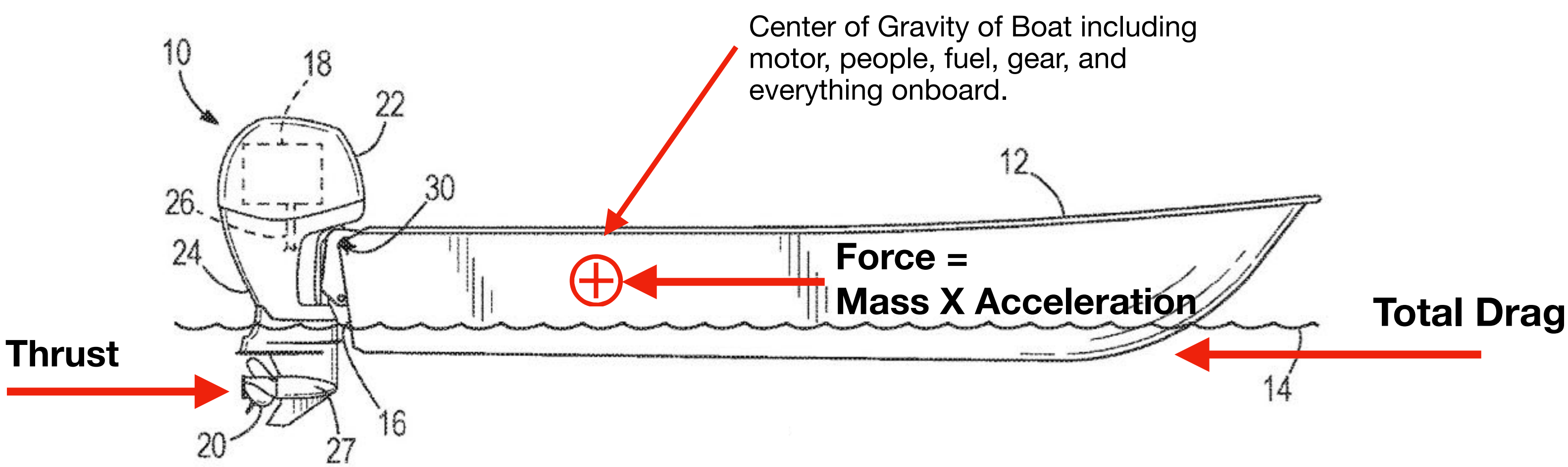
Very Basic Equations
& Time Constants

Introduction to the Boat Model

This paper is not going to get into complex mathematics sometimes used to model boats, however a very basic model including time constants will be introduced. The image below was cropped from a Brunswick patent and modified.

Figure 34: Forces to be Balanced When Boat is in a Top Speed (zero to max speed) Run



At take off, all propeller thrust goes toward accelerating the boat because Drag=0 (drag is created by speed and speed is zero). Between getting underway and top speed propeller thrust is split between accelerating the boat and drag. At top speed the boat is no longer accelerating, all propeller thrust is consumed by drag.

Some boat models recognize many kinds of drag. This model in **Figure 34** lumps them all into Total Drag. Similarly some models recognize an added mass of water the boat carries along with it. This model only recognizes mass of the vessel and its contents.

At takeoff, propeller thrust jumps from zero to a much higher value. Physics models refer to the jump as a step input.
Once the boat is underway, the boat stores energy as it begins to accelerate. If the engine was turned off, the boat would glide some distance under its own power as that energy was dissipated by drag.

The system in **Figure 34** has one energy storage element (the boat itself). Systems with only one energy storage element are called First Order Systems.

Methods described in this paper were used to curve fit the velocity curve during top speed runs. The coefficient **b** was representative of how fast the boat reached top speed.

Time Constant Tau

First Order Systems often use Tau to represent their time constant.

τ is typically used as the symbol for Tau

Using this methodology the Velocity equations used in this paper would be of the nature of:

$$V = A\,(1 - e^{-(1/Tau\,)t})$$

Tau has dimensions of 1/seconds or seconds^{-1} making the exponent of **e** dimensionless. The larger Tau, the slower the system responds. The smaller Tau, the faster the system responds.

In terms of the velocity equations developed in this paper, **b** = 1/Tau

Time constant Tau has more physical meaning than **b.** Somewhat like standard deviation of the bell curve, Tau is a standard measure of how quickly the variable changes. For example, tables show how fast the variable, such as velocity in our instance, increases. After one time constant the variable reaches 63.2% of its final value per **Figure 35**[10]. Similarly the curve reaches 86.5 percent in 2 Tau, 95% in 3 Tau, etc.

Some groups suggest the system has practically reached its max by 4 Tau (98.2%) while others suggest using 5 Tau (99.3%).

---

[10] Output of First-Order Step-Input Systems at Various Times. Introduction to System Dynamics. Shearer, Murphy, Richardson. Second Printing. April 1971. Pg. 126.

Figure 35

TABLE 5–4

Output of First-Order Step-Input Systems at Various Times

t	$v/V_f = 1 - e^{-t/\tau}$
0	0
τ	0.632
2τ	0.865
3τ	0.950
4τ	0.982
5τ	0.993
6τ	0.998

See Reference #13

Time Constant Example Using Displacement Mode Velocity in Figure 20

Figure 20 plotted the curve fit equation of boat velocity during displacement data mode along with the actual data points.

Figure 36 is a marked up version of **Figure 20** with multiples of Tau marked on the time axis. Since the take off point was .1846 seconds to the left of zero, that amount was subtracted from each multiple of Tau as calculated in the Tau table below.

b = .3351

Since b = 1/ Tau   Tau = 1/**b** = 1/.3351 = 2.98 seconds

Tau Table

1 Tau =   2.98 -.18 = 2.80

2 Tau =   5.96 - .18 = 5.78

3 Tau =   8.94 - .18 = 8.76

4 Tau = 11.92 - .18 = 11.74

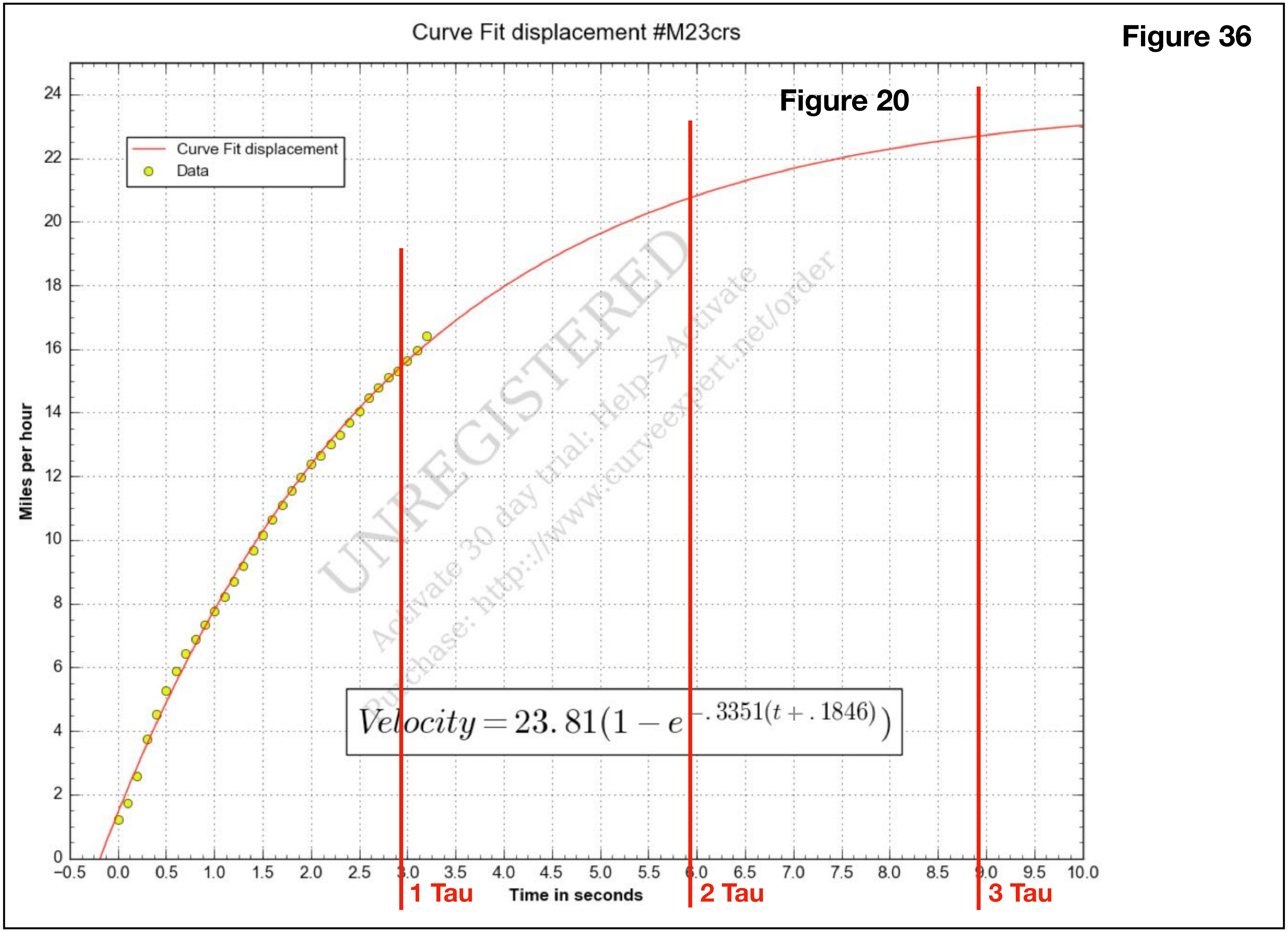
5 Tau = 14.90 - .18 = 14.72

Verifying the Value of Tau

1 Tau = 2.80 seconds on **Figure 36** because time starts .18 seconds to the left of zero.

I checked the value of Velocity at 2.80 seconds (1 Tau = 2.98 but started .18 seconds to left of zero) as calculated by Excel earlier as 15.05 mph. The actual velocity was 15.13

With a final velocity of 23.81 per the curve fitting program, 15.05/23.81 = 63.2 percent of the final value, the exact ratio as predicted in **Figure 32**.



Curve Fit displacement #M23crs
Figure 36
Figure 20
Curve Fit displacement
Data
Miles per hour
Time in seconds
1 Tau
2 Tau
3 Tau
Velocity = 23.81(1 − e^(−.3351(t + .1846)))
UNREGISTERED
Activate 30 day trial: Help->Activate
Purchase: http://www.curveexpert.net/order

Tau has Greater Physical Significance Than b

As previously shown:

1. Tau is often associated with First Order Systems with one energy storage element (is used in many other models).

2. Tau not only indicates how quickly velocity increases, at multiple values of Tau velocity can be calculated by only knowing maximum velocity.

3. Knowing the percentage of the final value of the variable (top speed in this instance) at multiples of Tau allows the curve to be sketched if maximum velocity is known.

4. Reference books consider the final value of the variable (top speed in this instance) to be reached at 4 or 5 Tau.

5. Just as the initial slope of the velocity curve was **b X A** where A is the final velocity (maximum velocity), **initial slope of the curve is A/Tau**.

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Conclusion

Summary

This paper presents a unique method of modeling recreational planing boats in top speed runs. Others are encouraged to duplicate the efforts of this paper (see Safety issues on **Page 99)** and share their results.

This method provides a way to represent displacement and planing portions of top speed runs with only 6 coefficients. Additionally, those 6 coefficients each have a direct impact on displacement and planing velocity curves.

The ability to convert velocity data per tenth of a second into displacement data per tenth of a second allows plotting total distance, propeller slip vs velocity and propeller slip vs time.

Similarly, changes per unit time in velocity data can be used to calculate acceleration. A smoothing equation improved the behavior of acceleration data. Acceleration data provides interesting insights into the transition zone when the boat moves from displacement mode to planing mode.

Overall this method provided unprecedented graphical insight into top speed runs.

The method provides a means of evaluating drag of the same vessel in various configurations.

While coast down data has yet to be presented (see **Appendix B** for a preview) it promises direct calculation of drag.

Review of Potential Applications of This Work

The applications below are POTENTIAL applications. These methods need to continue to be verified and checked against technologies currently available. The work currently looks very promising. Potential applications include:

1. Virtually racing top speed runs of vessels against one another or virtually racing the same vessel against itself after changes.

2. Comparing the drag or relative drag of a vessel with a propeller guard against the same vessel without a propeller guard.

3. Calculating drag or relative drag of a propeller guard at any speed using coast down data (see **Appendix B**)

4. Comparing drag or relative drag of outboard motors of the same or different manufacturers.

5. Reducing the amount of data to be stored from a top speed run from hundreds data points to just 6 coefficients.

6. Quickly comparing one top speed run against another is much more meaningfully than just looking at time to 20 mph, time to 30 mph, and top speed.

7. Physical meaning is directly associated with the coefficients. For example in terms of time to plane, the value of Tau for the displacement mode is extremely significant. How fast the boat can reach 63.2 percent of its top speed strongly correlates with time to plane.

8. Developing the full velocity curve from partial data runs, including coefficients, allowing prediction of top speed without actually running at top speed. This allows faster boats to be tested by less experienced operators (don't have to actually run to top speed) OR during mildly crowded lake conditions that do not allow safely running all the way to top speed OR on lakes without enough length to reach top speed.

9. Comparing the same boat with different horsepower drives while numerically showing the impact of horsepower changes on boat performance, including the effect of any changes in weight.

10. Demonstrating changes in performance by putting more fuel, people, or gear on the boatl.

11. Identifying time to plane.

12. Providing insight into the transition zone between displacement and planing mode.

13. Estimating top speed from boat velocity at 4 Tau or 5 Tau.

14. Estimating propeller thrust.

15. Estimating boat velocity during accidents will have application of the preview of our coast down paper. (see **Reference #6** and **Appendix B**).

16. Visibly illustrating how propeller slip is or is near to 100 percent at takeoff. Many think prop slip always ranges from about 8 to 15 percent. That is not true at takeoff and is a reason recirculation occurs in certain situations (the same fluid goes through the propeller, loops around, and goes through again and again). Recirculating is especially relevant to those in the water during take off in shallow water such as a houseboat backing off a beach.

17. Evaluating hull cleanliness on larger vessels. Cleaner hull has less drag.

18. In its most basic application, total coast down distance is a measure of relative drag after changes are made.

19. Investigating passenger stability (impact of velocity, acceleration, and jerk) on seated and standing passengers.

How This Work Came to Be

The innovations in this report came to be because five elements came together at a time a challenge was identified. The challenge was finding a better way of representing and analyzing top speed run boat test data.

Five things converged at that time:

1. Meeting Edmond R. Burke, U.S. Olympic Cycling Team Staff Member, at the Sports Equipment / Technology Meeting of the Sports and Technology Committee of the U.S. Olympic Committee at the U.S. Olympic Complex in Colorado Springs in 1984. I was introduced to Coast Down testing of bicycles during this event.

2. My work at Ditch Witch Trenchers resulting in the CMW Hydraulic Thermal Design Manual in 1983 brought me to the basic equation used for boat coast down studies.

3. My study of olympic weight lifting models led to the 1977 paper, Biomechanical Analysis With Cubic Spine Functions, by Thomas M. McLaughlin and others. That paper, in combination with my visits with Mr. McLaughlin led to the 9 Point Moving Chord Average method used to smooth velocity data when calculating boat acceleration.

4. A MerCruiser technician referred to as Steve in this paper who took an interest in the project and cheered its progress.

5. Availability of spreadsheets incorporating macros and availability of complex curve fitting software.

The Importance of Safety

This paper is NOT professional advice.

Anyone attempting to replicate testing of the nature discussed in this report does so at their own risk.

Serious attention should be addressed to all safety issues involved in collecting such data, especially for faster vessels. Such as:

1. Use of experienced boat operators
2. Well maintained boats with all required safety gear onboard
3. At least two people onboard, one to operate the boat and one to operate instrumentation
4. A large non-crowded lake allowing long straight runs at speed
5. Use of a chase boat at higher speeds
6. Attention paid to cold water (hypothermia), communications with a base station elsewhere, someone making sure they make it back after each trip to the lake (like filing a flight plan), etc.
7. Always wearing life jackets and keeping kill switch cords attached
8. Beginning each new vessel tested with slower runs to acclimate boat operators and technicians to the vessel and conditions
9. Use of helmets when appropriate
10. Make sure everybody involved knows how to swim
11. Make sure your chase boat has a means to recover rescued individuals from the water (such as a swim platform or man overboard recovery system)
12. Consider use of a hinged flap behind the shift-throttle control. The hinged flap flips over and lays flat across right behind the throttle in neutral. When flipped over, it prevents the shifter from being pulled into reverse. When top speed is reached and the shift-throttle control is rapidly pulled back to neutral, the flap prevents the shifter from being pulled back all the way into reverse by error.
13. All other safety precautions necessary for the conditions and vessel(s) being tested.

The Future

As mentioned earlier some coast down testing was done using these same methods, see **Appendix B**. The coast down study provides considerable insight into vessel drag at various speeds and modes. We hope to present that information in greater detail in a future paper.

Use of slip ring thrust measurement devices to directly measure thrust or approaches like those in Brunswick's U.S. Patent 5,527,194 could allow collecting real time thrust data for comparison with these findings and with the yet to be published coast down data.

Use of modern techniques to capture boat velocity multiple times per second (GPS, Differential GPS, inertial navigation, Sanshin/ Yamaha U.S. Patent 4,956,977, etc.) with greater accuracy during peak acceleration and peak deceleration periods.

Direct capture of acceleration via an accelerometer.

Feedback

Please let us know what you thought about this paper, if you were able to use these techniques to collect and analyze your own data, and if you have any tips to share about the methods described here.

You can contact us from the **Contact Us tab on the top menu at PropellerSafety.com**

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Appendix A

Calculating Displacement from Discrete Velocity Data

Calculating Displacement From Discrete Velocity Data

Many methods are used to estimate displacement from discrete velocity data. Among the first decisions is whether to smooth velocity data before processing it or not. The macro used in this paper processed non-smoothed velocity data to determine displacement. The option to smooth the velocity data and rerun the data was then presented. A few runs in rougher water were smoothed but most of the top speed runs were not smoothed.

Mercury's portable data logger system used the most recent velocity data point and calculated incremental displacement as if that velocity had been in effect over the entire previous 1/10th of a second. The same approach was used in this paper. See **Figure A-1** and **Figure A-2**.

For example if velocity was zero at time =0, velocity was 1 mile per hour at 1/10 second, velocity was 2 miles per hour at 2/10 of a second into the top speed run, 3.5 miles per hour at 3/10 of a second and 5 miles per hour at 4/10 second the data points would generally look like **Figure A-1.**

Displacement or position is the integral of velocity vs time, meaning the area under the velocity time curve.

The method chosen to integrate displacement data is shown in **Figure A-2**. As mentioned earlier, the most recent velocity data point was used as the average velocity over the last time step.

Figure A-3 shows how this method might run into problems. If the velocity is significantly increasing each tenth of a second, the area above the blue line is being added to each segment when it should not be added.

It would be more proper to use the average velocity over the last time step than the velocity at the end of the last time step. However, the percentage increase in boat velocity over each tenth of a second was quite small in comparison with current velocity. Thus the sliver of displacement we added (the area above the blue line) was quite small. For example runs of over 3000 feet were found to only have a 5 foot discrepancy with displacements calculated using the more accurate velocity averaging method. Thus we elected to use this method due to its simplicity and due to it previously being used by Mercury.

For example in the top speed run used for many of these charts the boat only travels about 7.5 feet in the first full second of the run. The more complex integration method has it at 7.26 feet while our simplified method has it as 7.83 feet. Thus total travel was under estimated by about .61 feet. As the boat continues to speed up, the incremental change in velocity becomes less and less. Thus at 40.4 seconds into the run the delta between the two methods only grew to 5.71 feet. A faster accelerating boat would have even less difference between the two values.

During top speed runs the boat is constantly accelerating til it reaches maximum speed. Thus inside each tenth of a second during the early and mid portions of the run the boat spends more time near the speed at the beginning of the tenth (is going slower) than at the speed near the end of the tenth (is going faster). This slightly compensates for the decision not to average the velocities.

Figure A-1

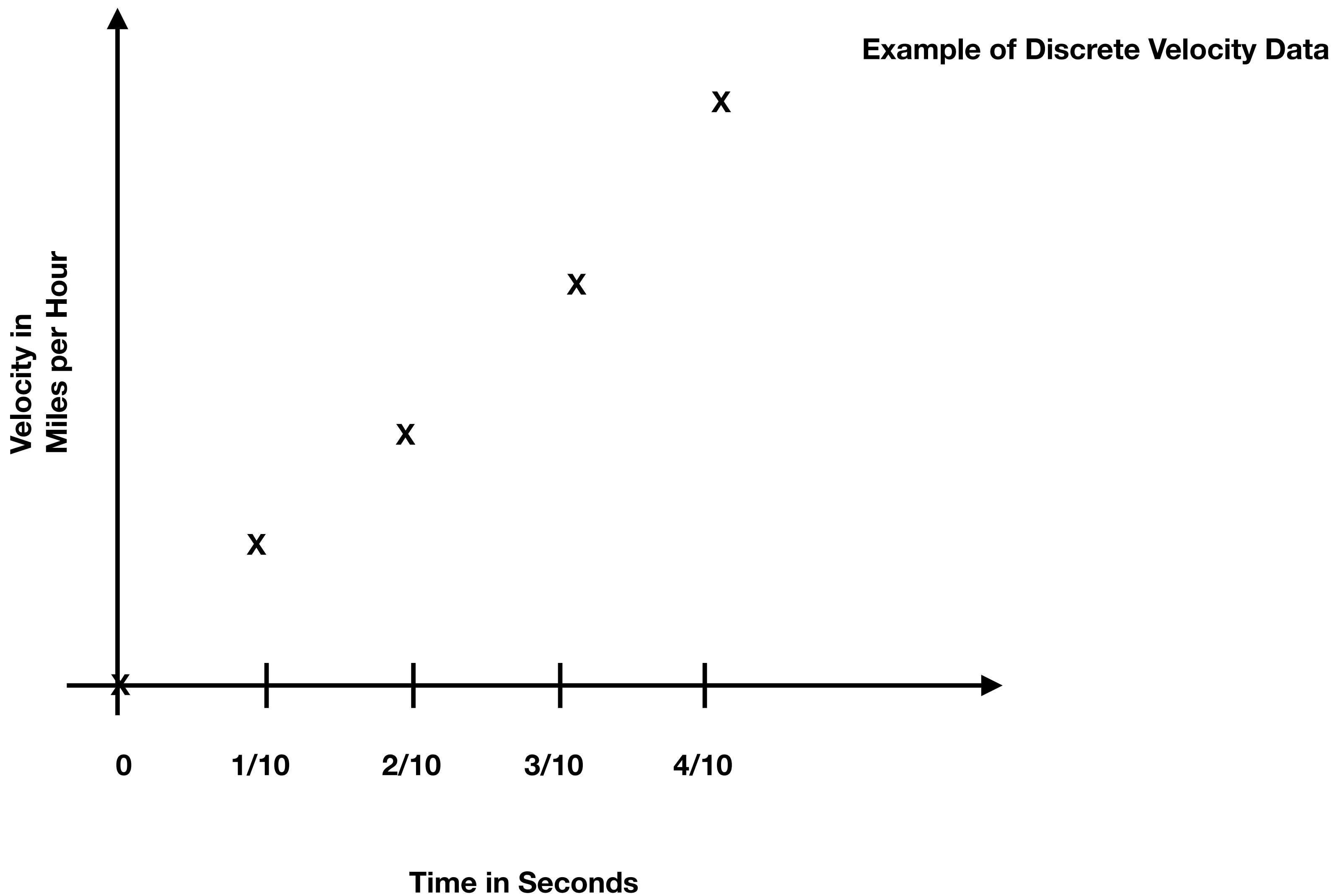

Example of Discrete Velocity Data
Velocity in
Miles per Hour
0
1/10
2/10
3/10
4/10
Time in Seconds

Figure A-2

Calculating Displacement

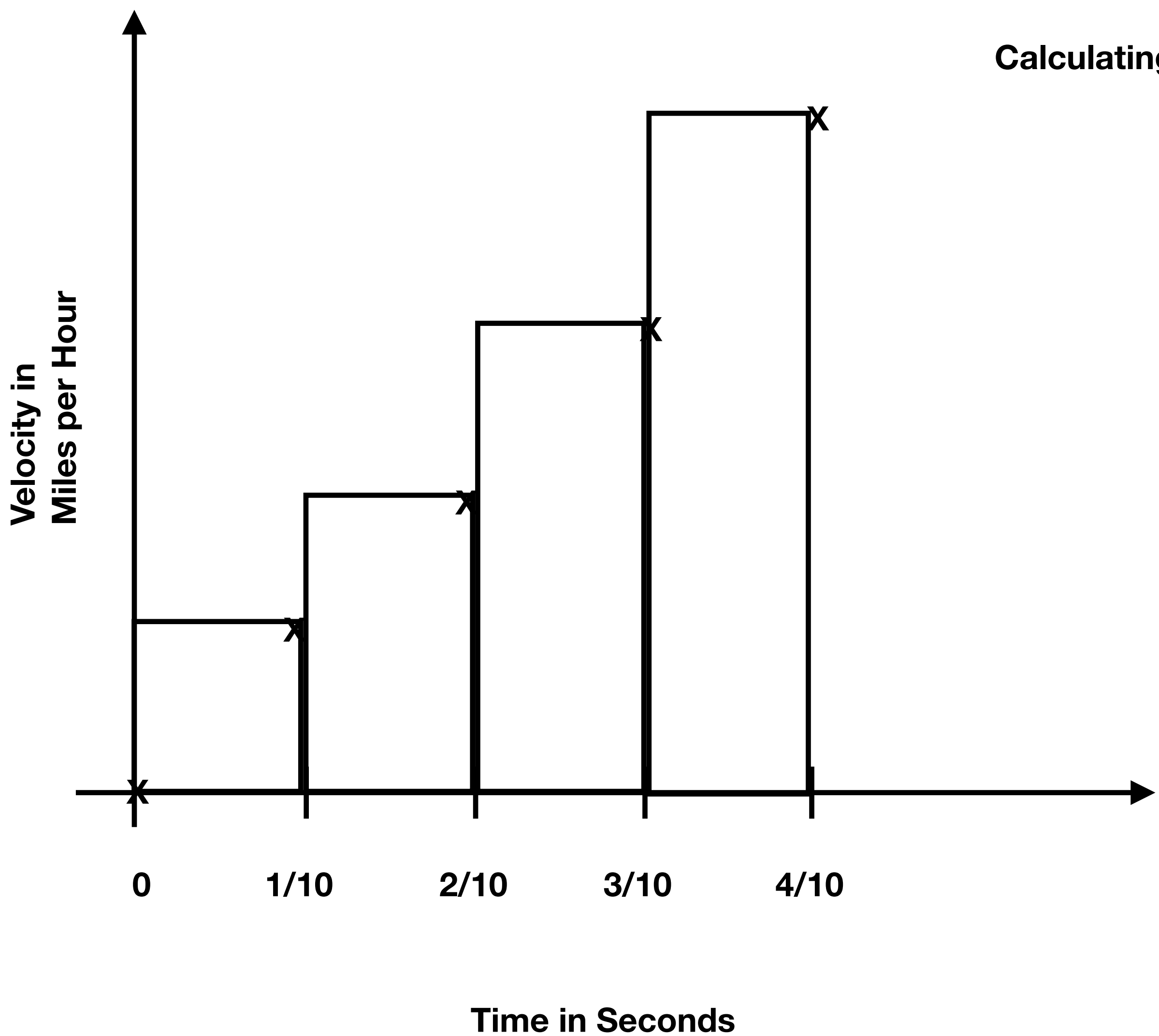
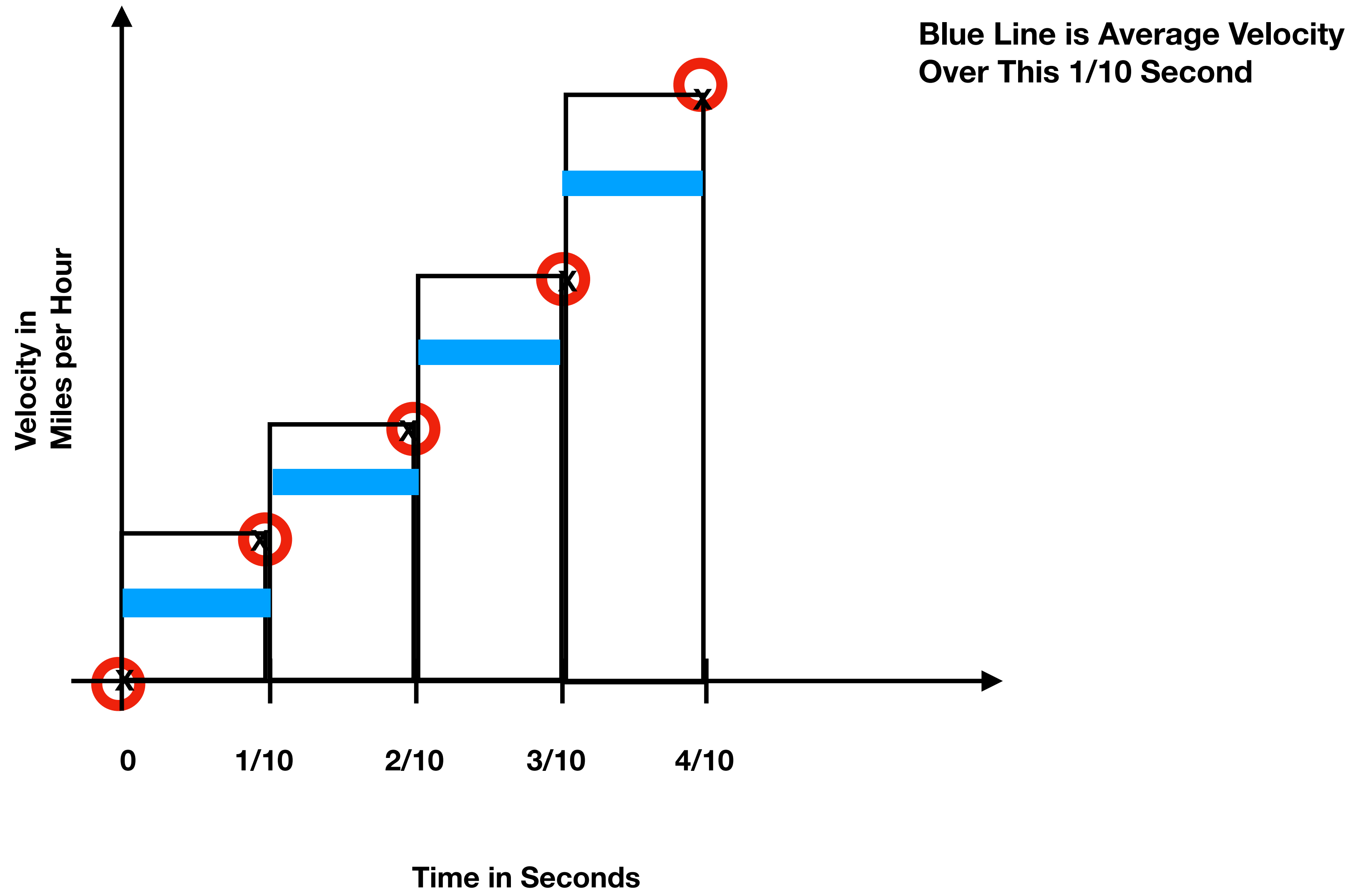


Figure A-3


Blue Line is Average Velocity
Over This 1/10 Second
Velocity in
Miles per Hour
X
X
X
X
X
0
1/10
2/10
3/10
4/10
Time in Seconds

Blank Page

Appendix B

Preview of Coast Down Paper

Coast Down Preview

Appendix B previews a coast down paper we hope to publish in the future. As mentioned earlier, when a recreational planing boat is running in planing mode at top speed, the boat is no longer accelerating. All the propeller thrust goes into overcoming drag. If the shift throttle control is thrown to neutral at that time, the boat begins to coast back down to rest.

Figure B-1 shows a full run from take off, to planing, to top speed, to coast down planing mode, to coast down displacement mode, to being at rest.

Curve Fitting Coast Down Curves

Similar to the acceleration portion of the top speed run, velocity data for the deceleration portion of the top speed run is curve fit to:

$$V_{pcd} = A_{pcd}(e^{-b_{pcd}(t-t0_{pcd})})$$

Where the subscript **pcd** refers to the planing coast down portion of the curve.

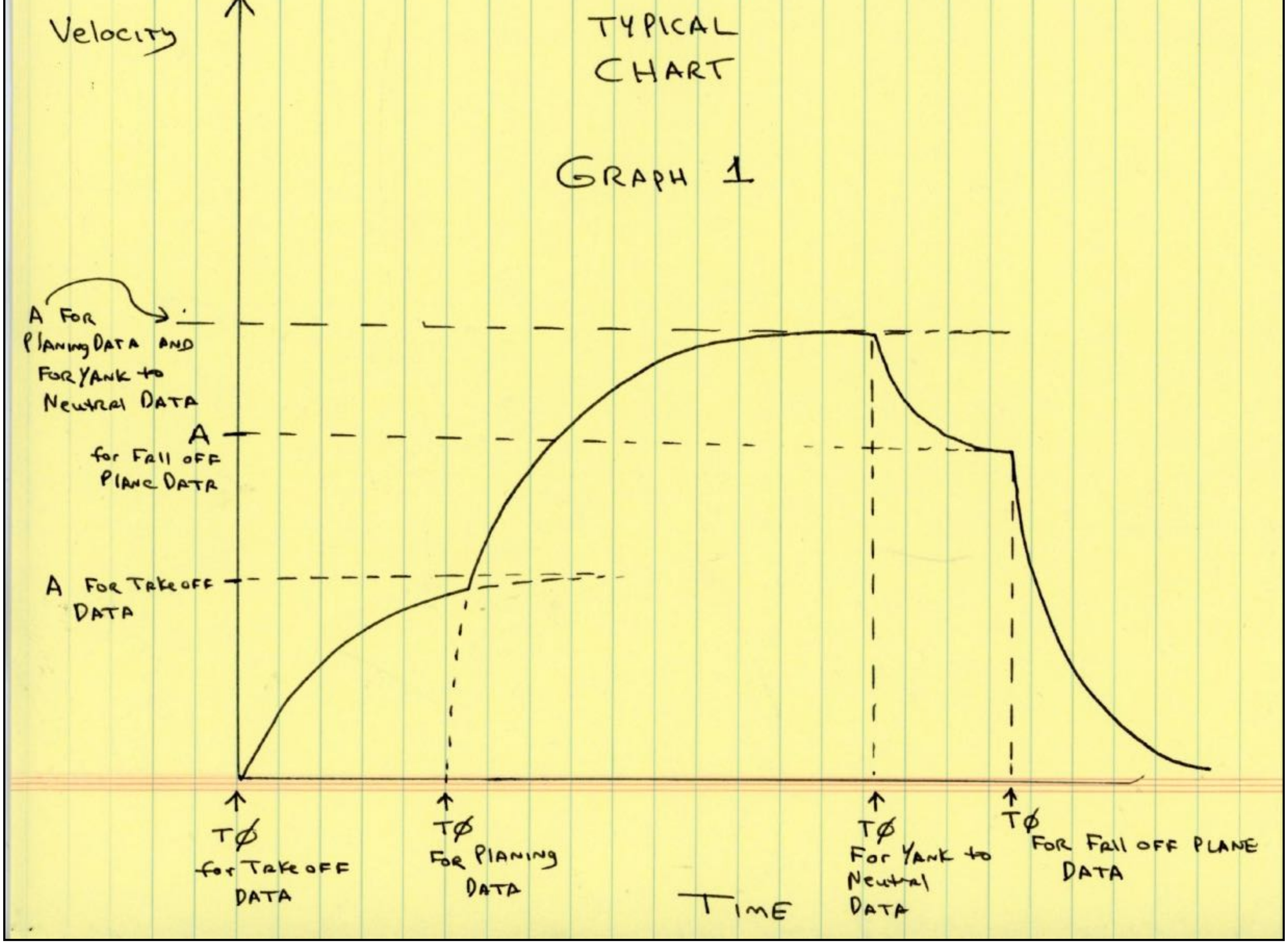
$t0_{pcd}$ represents the time at which the shift-throttle control was thrown to neutral.

Earlier **Figure 3** showed a top speed run for a tunnel drive. **Figure B-2** shows the rest of the run in which the boat coasted down to rest, all four sections of the curve were curve fit, two of which previously curve fit in this paper.

RPM & Velocity Chart, Figure B-3

The RPM & Velocity Chart earlier presented as **Figure 4** is repeated here as **Figure B-3**. Note RPM/100 is plotted on the left axis so it can be shown in the same range as velocity. The deceleration portion of the run begins at about 20 seconds into the run.

Figure B-1: Generic Full Run Chart



PART 1 $V = 28.243\left(1 - E^{-(.231)(T-(-.248))}\right)$

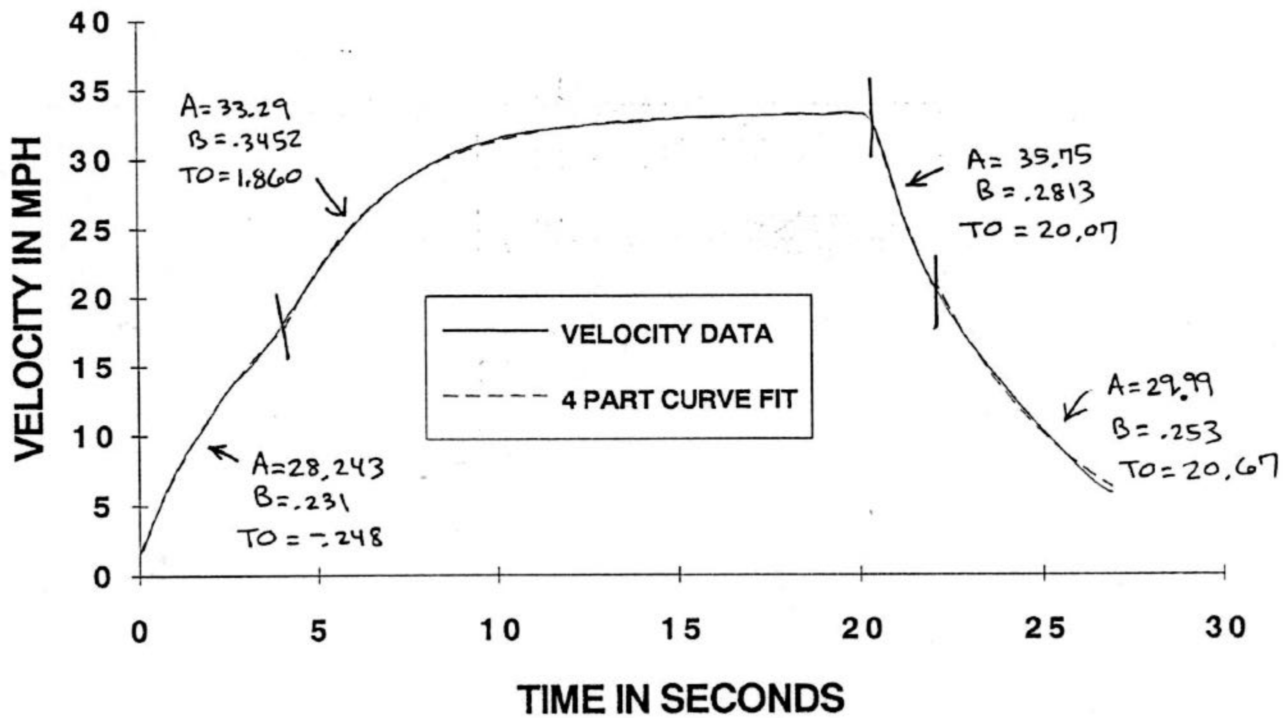
GRAPH 3

PART 2 $V = 33.29\left(1 - e^{(-.3452)(T-1.860)}\right)$

PART 3 $V = 35.75\left(e^{-.2813(T-20.07)}\right)$

PART 4 $V = 29.99\left(e^{-.253(T-20.67)}\right)$

Figure B-2: 4 PART CURVE FIT OF VELOCITY DATA



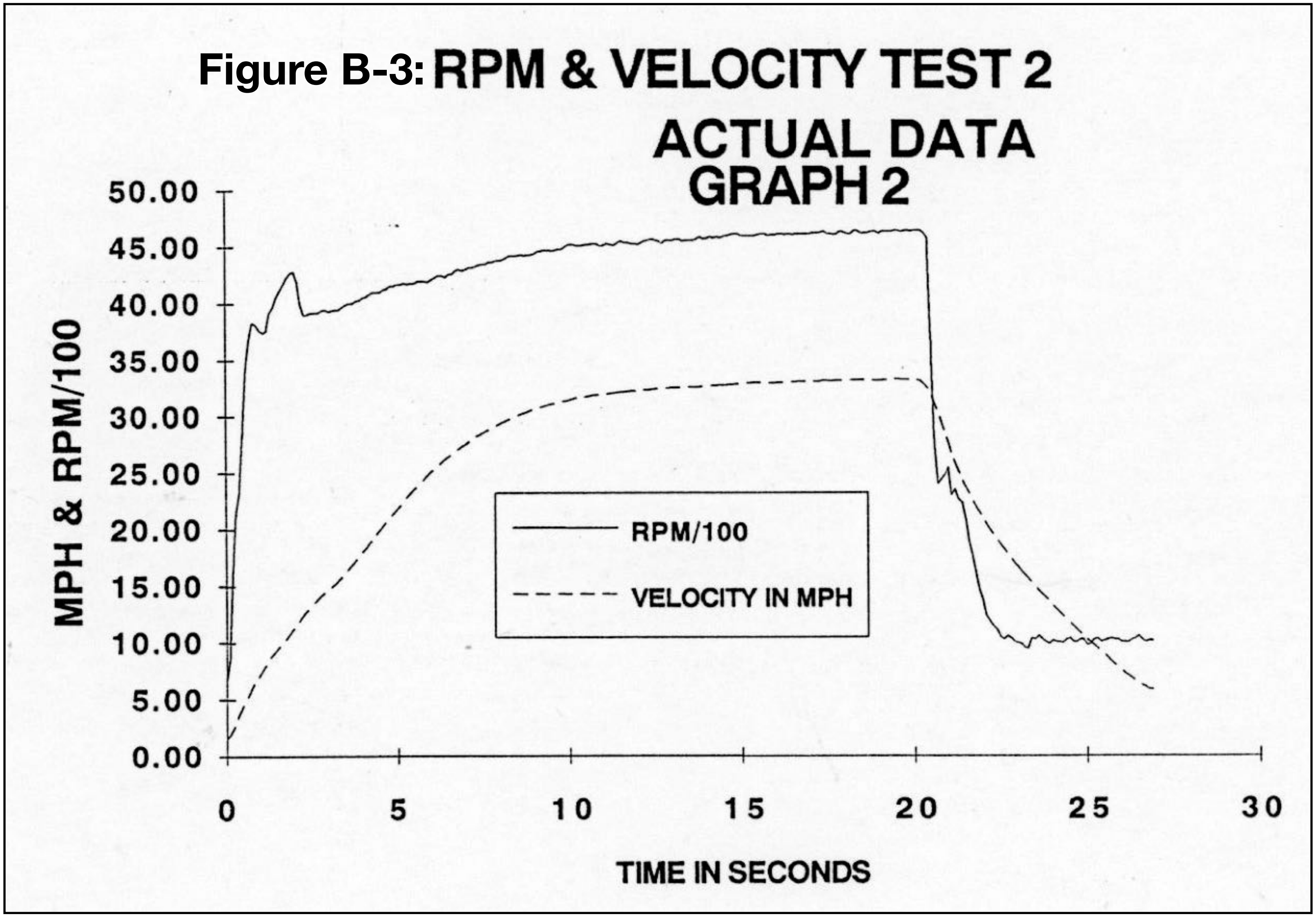


Figure B-3: RPM & VELOCITY TEST 2
ACTUAL DATA
GRAPH 2
50.00
45.00
40.00
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
MPH & RPM/100
RPM/100
VELOCITY IN MPH
0
5
10
15
20
25
30
TIME IN SECONDS

Full Acceleration - Full Deceleration Chart, Figure B-4

An example of a boat accelerating from rest to top speed, then coasting down to rest is shown in **Figure B-4**.

Two acceleration curves are shown, one very closely overlapping the other. Acceleration was calculated directly from velocity data AND from the equation curve fit of the velocity curve. Both representations are quite similar.

There is a noticeable discontinuity when the boat goes from displacement mode to planing mode at about 4 seconds into the run. Then another discontinuity at about 20 seconds into the run when the boat throttle is quickly pulled back to neutral. The last discontinuity at about 22 seconds into the run may be the decelerating boat falling off plane and going into displacement mode or it may just be a flyer (bad data point).

Force = Mass X Acceleration

Thus with a fixed mass (like the boat and its contents) Acceleration is directly proportional to Force.

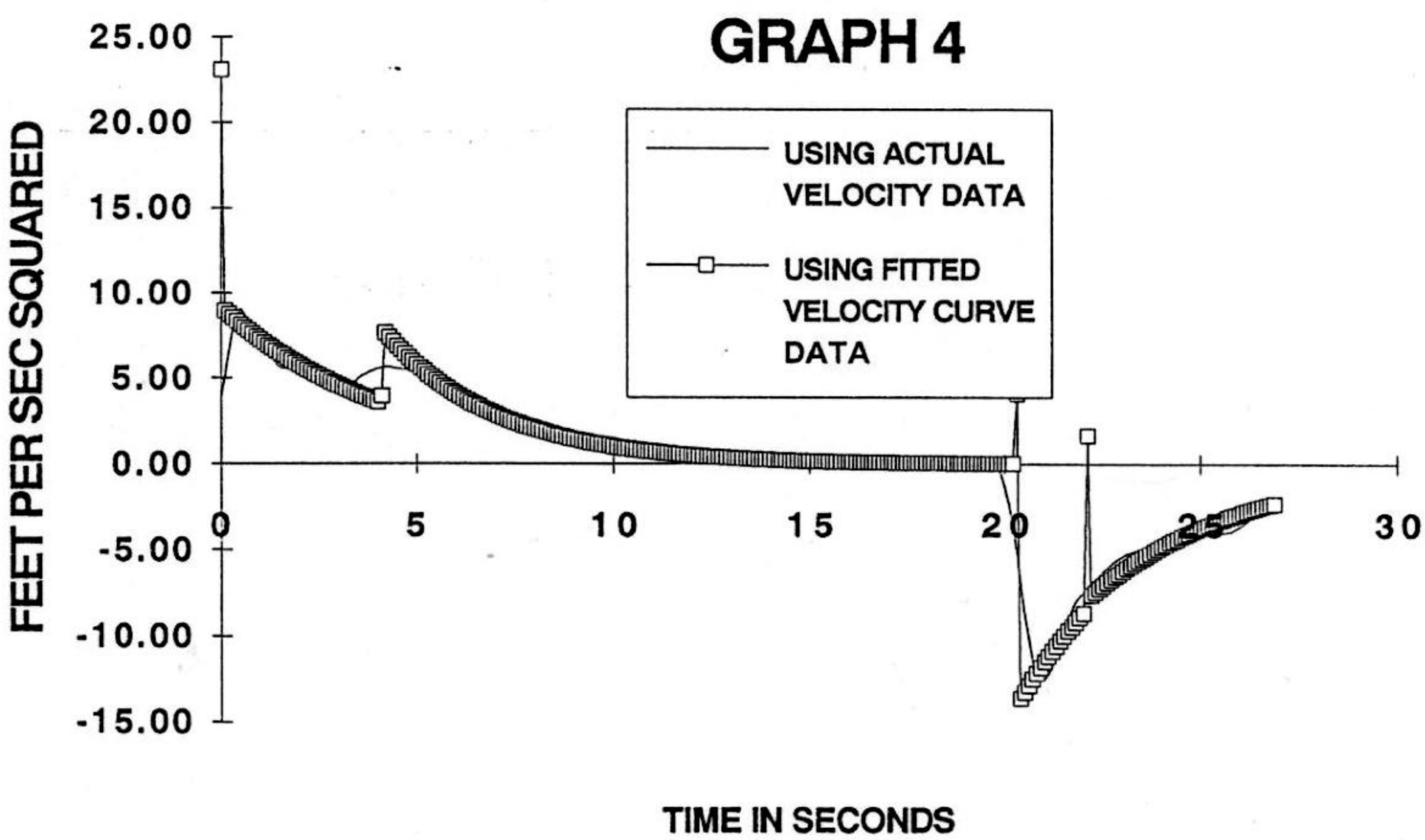
Acceleration is proportional to total thrust at takeoff. Once the boat gets up and going some force goes into accelerating the boat and some force goes into overcoming drag.

At takeoff, Force going toward accelerating the boat is at its maximum positive value. Force going to accelerate the boat rapidly decreases til the boat goes on plane at about 4 seconds. The force available to accelerate the boat significantly jumps when the boat goes on plane because drag is significantly reduced. The magnitude of the jump in acceleration from displacement mode is directly proportional to the difference in planing mode drag vs displacement mode drag when the boat jumps up on plane.

When the boat reaches top speed, acceleration goes to zero. Once the shift-throttle is thrown to neutral, the largest magnitude of deceleration is encountered. Then the boat slows, falls off plane, and eventually comes to rest once again with zero acceleration.

In **Figure B-4**, two thin lines are seen crossing over most discontinuities. The thin line that goes directly from one end of the acceleration curve to the next end after the discontinuity is the acceleration curve based upon curve fitting the velocity curve. These values are calculated from that equation (actually from the slope of that equation). The other thin line leaves the acceleration line a few points before the discontinuity and rejoins on the other side a few points past the discontinuity. This thin line comes from the 9 Point Chord Average method being applied to raw velocity data. It takes a few steps for the equation to start generating data points.

Figure B-4: ACCELERATION TEST 2



Determining Acceleration During Coast Down

Acceleration is the slope of the velocity curve. Similar to before, velocity curve slope can be estimated 5 ways:

1. Graphically - plot Velocity vs Time data of the coast down portion of the run and use a protractor to estimate its initial slope,

2. Calculate slope - use data during the first few tenths of a second after coast down begins to calculate slope.

3. Method #2 above except an algorithm is used to smooth velocity data during the coast down process of calculating slope during the first few tenths of a second of coast down in planing mode.

4. Derivatives - use the three steps below
   A. Mathematically curve fit coast down curve planing portion data (see **Figure B-1**),
   B. Mathematically take the derivative of that equation (see the next section),
   C. Evaluate that derivative at time = $\mathbf{t0_{pcd}}$ to determine slope of the velocity vs time curve when coast down begins.

5. Use Tau, the time constant (see **Figure 35** and **Figure 36**) and maximum velocity. **b = 1/Tau**. Initial slope = Maximum Velocity / Tau for each mode (displacement mode and planing mode).

This paper will focus on the Derivative Method, method #4 above.

Curve Fitting Planing Coast Down Velocity Data

The equation to be curve fit during coast down is reproduced below:

$$V_{pcd} = A_{pcd}(e^{-b_{pcd}(t-t0_{pcd})})$$

Mathematically Taking the Derivative of the Planing Portion of the Coast Down Velocity Curve

Differentiating the equation above

$$dV_{pcd}/dt = \text{acceleration} = A_{pcd}(-b_{pcd})e^{-b_{pcd}(t-t0_{pcd})})$$

evaluate derivative at time = **$t0_{pcd}$** to determine initial slope

$$(dv_d/dt) \,| = -A_{pcd}b_{pcd}e^{-b_{pcd}(0)} = -A_{pcd}b_{pcd} = \text{Acceleration}$$

slope = negative Maximum Velocity in planing mode X b_{pcd} or -Vmax/Tau_{pcd}

The displacement portion of the coast down run can similarly be curve fit and differentiated.

Calculating Top Speed Drag

At the moment the shift-throttle control is thrown to neutral at top speed, **Force = Mass X Acceleration**

Force at that moment = Drag at top speed

Thus Drag at top speed = Mass X Acceleration at the moment deceleration begins.

Note acceleration will have a negative value because the boat is decelerating.

As seen on the previous page, Acceleration at the moment coast down begins is equal to **$- A_{pcd} b_{pcd}$**

Therefore **Top Speed Drag = - Mass X Maximum Velocity X b_{pcd}**

Applying Top Speed Drag Equations to Figure B-4

F = M X A   Mass = 2475 pounds/32.2 = 76.86 slugs

The peak of the smoothed deceleration was 12.59 ft/sec^2

F = 12.59 X 76,86 = 967.7 pounds = Drag at top speed.

Similarly, peak deceleration from point to point velocity data was 14.08 ft/sec^2

F = 14.08 X 76.86 = 1082.2 pounds = Drag

Thus the two methods estimate top speed drag at 968 pounds and at 1083 pounds, with the upper value being about 12 percent greater than the lower one.

Calculating Drag Through the Entire Coast Down

Once the shift-throttle control is thrown to neutral, propeller thrust goes to zero (or very quickly goes to zero) and the propeller windmills as the boat coasts down. With no thrust from the propeller, drag is the only force acting on the vessel. Thus drag is the only force slowing the vessel. Therefore drag can be calculated at every single velocity data point as the boat slows.

F= M X A

Boat weight with people, fuel, and gear can be used to determine drag using the acceleration calculated during coast down.

Figure B-5 shows a boat coasting down from about 33 miles per hour with the resulting drag force plotted each tenth of a second.

Note - time goes left to right on the x-axis as the boat gradually slows down.

When the boat falls off plane at about 10 miles per hour drag significantly increases (velocity decreases much faster) making the deceleration curve steeper.

Note - drag is normally considered as a negative force. It is just shown as a positive value in **Figure B-5** to better display the finer parts of the curve.

The chart of Drag vs Velocity curve (**Figure B-5**) viewed from left to right shows the boat having less drag as it goes on plane, then at about 22 miles per hour drag assumes a constant upward slope that only ends when all the power from the propeller is consumed by drag at about 33 miles per hour.

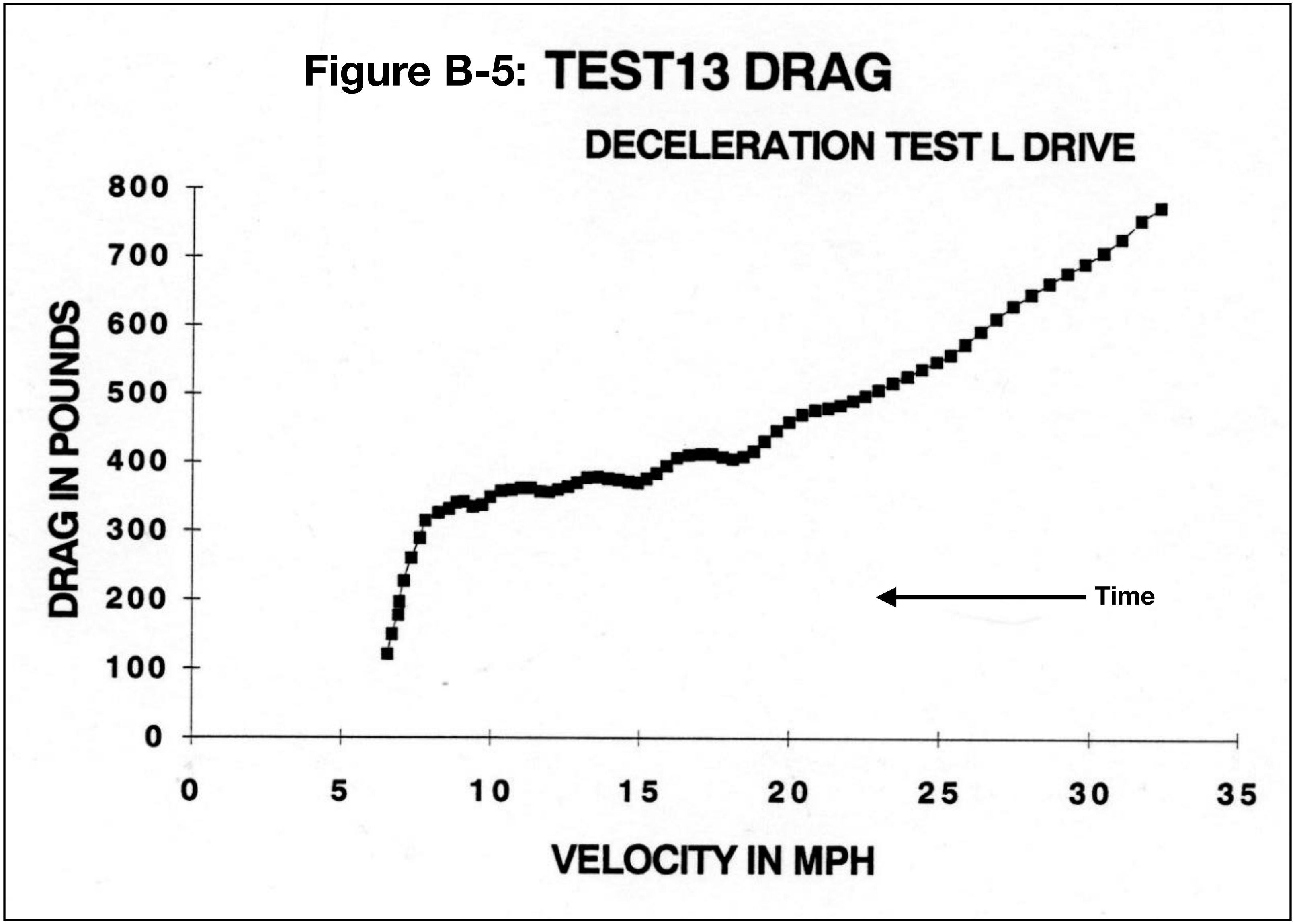


Figure B-5: TEST13 DRAG
DECELERATION TEST L DRIVE
DRAG IN POUNDS
800
700
600
500
400
300
200
100
0
0
5
10
15
20
25
30
35
VELOCITY IN MPH
Time

Comparing Drag of One Vessel vs Drag of Another Vessel

Two boats with different drives in them are shown in **Figure B-6**. Each boat makes two runs.

Note - **Figure 6** charts acceleration which is directly proportional to force. Prior to coast down the force plotted is the portion going toward accelerating the boat. Additional propeller thrust is going toward overcoming drag. During coast down, acceleration is proportional to total vessel drag.

The runs show all modes: take off, displacement mode, transition zone, planing mode, top speed, planing coast down mode, the transition from planing to displacement coast down mode, displacement mode coast down, and the boat slowing on down approaching being at rest.

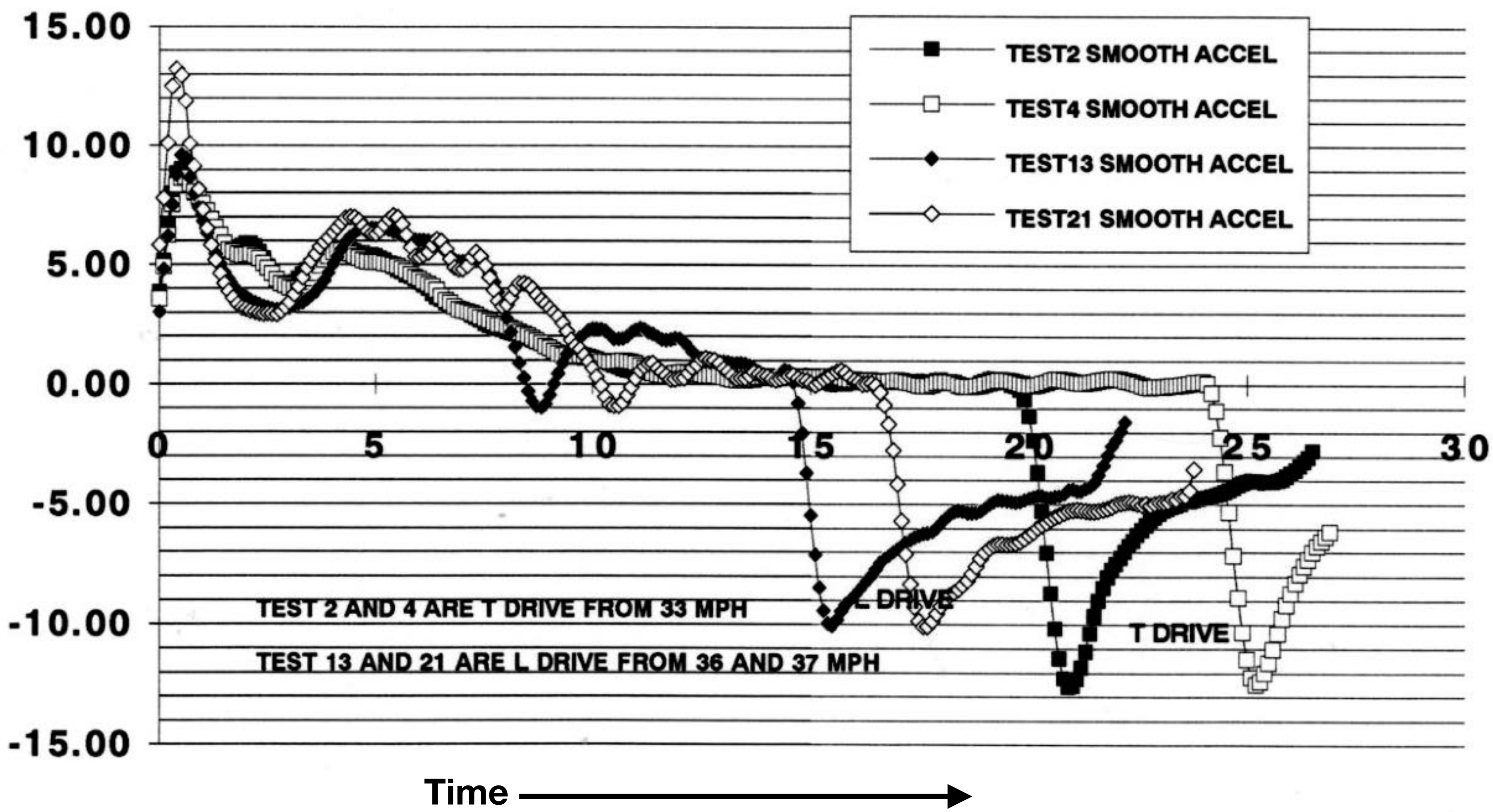
Test 2 and Test 4 are one type of marine drive. Test 13 and Test 21 are another type of marine drive in a different boat.

All acceleration data is calculated from 9-Point Chord Moving Average of the corresponding velocity data.

The two runs for each marine drive type appear very consistent except for one run having a higher acceleration at takeoff. They reached similar top speeds, had similar acceleration patterns, and coasted down similarly.

One drive type had a greater acceleration after going on plane than the other. All four runs reach top speed at about 15 seconds into the run when Acceleration = 0.

Figure B-6: SMOOTHED ACCELERATION T DRIVE VRS L DRIVE



Propeller Thrust vs Velocity Curve

The portion of propeller force that goes into accelerating the boat can be calculated for every tenth of a second during the run up from rest to top speed from **F= M X A**.

Similarly total drag can be calculated for every tenth of a second during coast down.

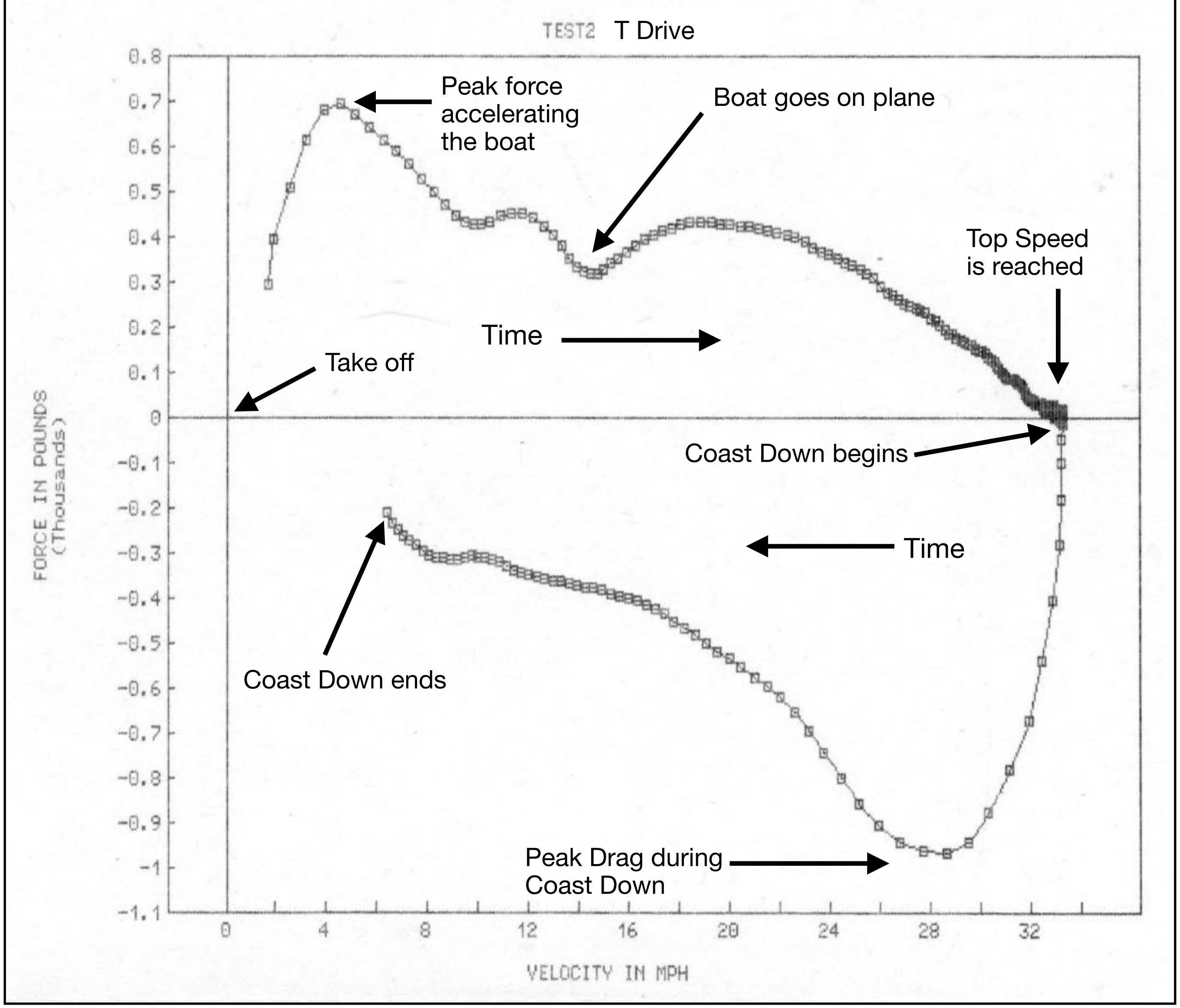
If force is shown to be positive during acceleration and negative during coast down, curves like **Figure B-7** are produced. This particular chart was produced from data acquired during Test 2 in **Figure B-6**.

Figure B-6 shows maximum force going to get the boat up on plane, then the amount of force going to drag increases as acceleration goes to zero.

When the shift-throttle control is thrown to neutral at top speed, the boat quickly encounters maximum drag. The boat is thrown to neutral at about 32 seconds and maximum drag is recorded at about 28.5 seconds. Part of the reason for the time delay is slow response of the pitot tube speedometer to abrupt changes as discussed earlier on **Page 35**.

Then the boat slows down and the lower portion of the data below 0 on the Y axis is laid down from right to left.

Figure B-7: FORCE VRS VELOCITY



Plotting Propeller Thrust

Figure B-7 and the data behind it allows plotting propeller thrust.

If you assume coast down force (drag) is the same as the amount of drag the boat faced going up to top speed, you can just add the portion of the propeller force going toward accelerating the boat (force in top half of chart) plus the drag (bottom half of chart) for each tenth of a second.

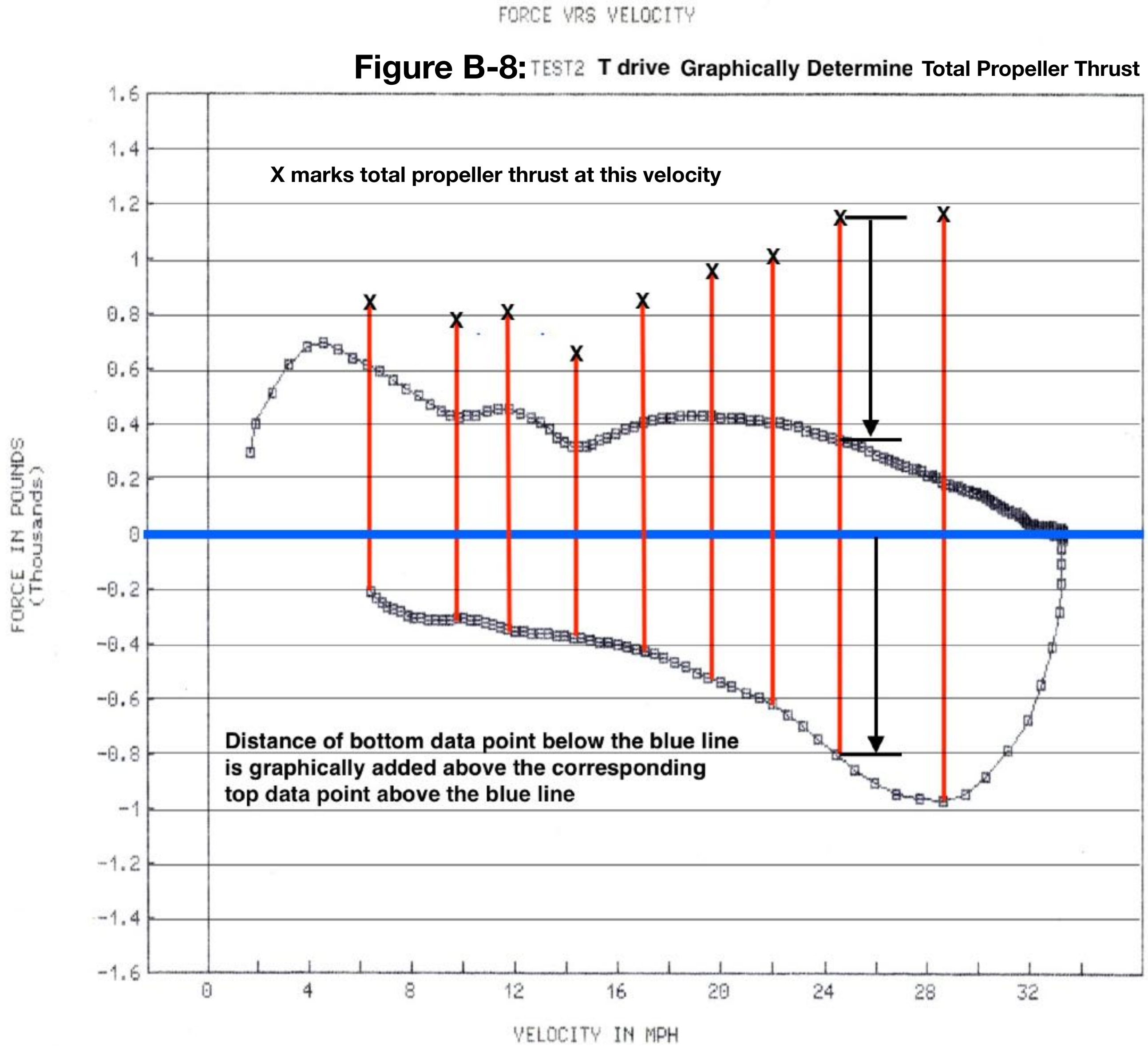
The forces can be graphically added such as in **Figure B-8**. The Y Axis value of data points below the blue line was added to the height of the upper curve. The assumption is that coast down drag at a given velocity equals the drag the boat was facing at that same speed as it was accelerating. While this may not exactly be true, it is close enough to get a general feel for total drag.

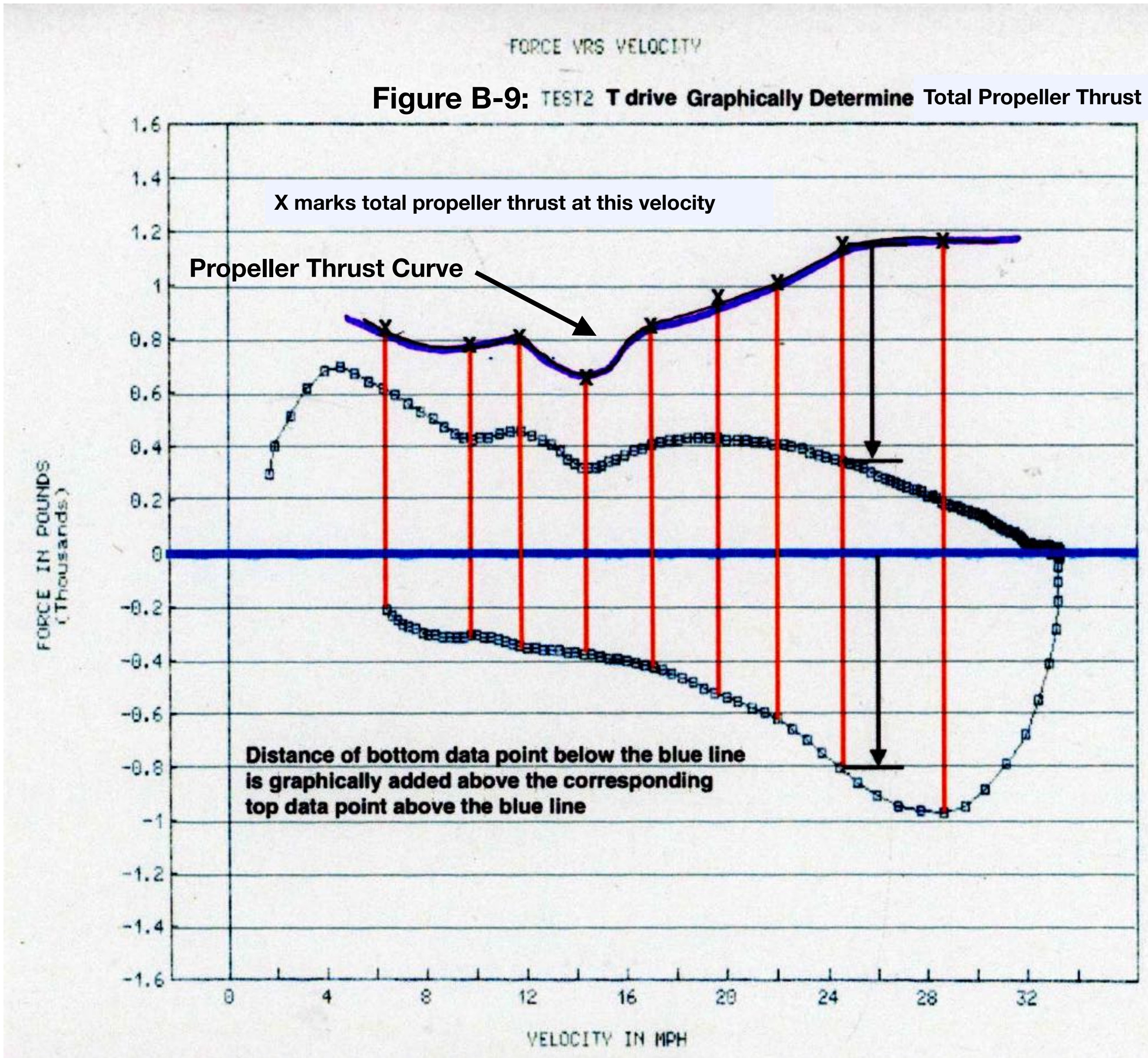
Values were graphically added as seen by the vertical arrows near the right side of the chart. The bottom arrow shows the bottom value is about four grid lines below the horizontal blue line (force axis). That same distance, 4 grid lines, was added to the corresponding upper data point as seen in **Figure B-8**. In this instance the lower vertical arrow was copied and placed above the upper data point to accurately place the total force data point. This process was repeated for each of the vertical red lines. The vertical red lines are not accurately spaced along the horizontal axis. They were placed in certain positions to bring a relatively consistent distance between them, plus to include dynamic areas of the curves.

While not shown here, multiple boats were coasted down from top speed, then coasted down from 5 miles per hour increments below top speed to compare the data. For example, if the boat ran 50 miles per hour at top speed, it was coasted down from 50 mph. Then it was also coasted down from 45 mph, 40 mph, 35 mph, 30 mph to see if the drag values determined at 30, 35, 40, 45 mph were the same as the drag values read at those speeds from the 50mph coast down. In general the values were the same. However, the first second or two of data was not accurate due to slow response of the pitot tube velocity pick up.

Figure B-9 sketches a curve over the total propeller thrust data points. The dip in the total propeller thrust curve at about 12 mph may be due to the boat going on plane, the load on the engine decreasing, and the engine revving up.

Figure B-8: TEST2 T drive Graphically Determine Total Propeller Thrust




FORCE VRS VELOCITY
Figure B-9: TEST2 T drive Graphically Determine Total Propeller Thrust
X marks total propeller thrust at this velocity
Propeller Thrust Curve
Distance of bottom data point below the blue line
is graphically added above the corresponding
top data point above the blue line
FORCE IN POUNDS
(Thousands)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
0
4
8
12
16
20
24
28
32
VELOCITY IN MPH

Confirming the Propeller Thrust Curve Shown in Figure B-9

Purely by definition Horsepower = Force X Velocity

See these two references, Horsepower in Drag,[11] and Aircraft Propeller Aerodynamic Process[12] for converting velocity, drag, and propeller slip to horsepower.

Horsepower = (Drag force in Pounds X Velocity in Feet per second)/((550 pound-feet/second)/horsepower)

Looking back at **Figure 34**, we see that total drag plus the force going into accelerating the boat = thrust

When the boat is at top speed it is no longer accelerating, all thrust is going toward overcoming drag.

The outboard used in the boat shown in **Figure B-8** and **Figure B-9** had a rating of 113 horsepower at the propeller shaft.

Two different top speed runs of the same boat showed a propeller slip at top speed of 14 percent.

That means the propeller shaft horsepower was diminished by 14 percent before the thrust was generated.

113 propeller shaft horsepower X (100 -14) percent = 97.2 horsepower

To envision coast down force at the moment the drive was thrown to neutral, think of the negative force increasing quicker than it does in the smoothed acceleration curve shown in **Figure B-9**, then rounding up to join the rest of its curve as the boat continues to slow down. With that in mind, the propeller thrust curve (**Figure B-9)** would show about 1,140 pounds of total thrust at a top speed of 33 miles per hour.

Going back to our horsepower equation of

Horsepower = (Drag force in Pounds X Velocity in Feet per second)/(550 pound-feet/second)

97.2 horsepower = 1140 pounds X (33 miles/hour) X (1 hour/3600 seconds) X (5280 feet/ mile) / (550 pound-feet/second)
97.2 horsepower = 100.3 horsepower

---

[11] Horsepower in Drag. Jim Russell of AeroMarine Research. Powerboat & RIB. November/December 2019. Pages 138-140.

[12] Aircraft Propeller Aerodynamic Process. Acronautics Guide. https://www.aircraftsystemstech.com/p/propeller-aerodynamic-process-airplane.html viewed 2 October 2020.

There is reasonable agreement between the two calculations.

While the methods presented in this paper may not be totally accurate, their lack in accuracy is made up for by the insight they provide into the dynamics of the boat.

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Appendix C

Boat Testing Example

How Some Manufacturers Present Top Speed Run Data in the Media

Many boat manufacturers, marine drive manufacturers, boating publications, and online boat test sites such as BoatTest.com supply boat performance test data. Top Speed run data is presented in numerous formats, the most basic which is just listing maximum speed.

Ranger presents some top speed runs in video format on YouTube, such as this 2018 Ranger Z520L[13]. The video is a blend of Mercury Vessel Views data and what looks like GoPro gauges. This particular run provides minimal details on the weight or running weight of the boat and does not directly record elapsed time in fractions of a second. Time is indirectly available from the video time line or by counting video frames.

Mercury Marine Boat House Bulletins tend to more thoroughly record boat specifications and test conditions along with top speed, RPM at top speed, time to 20 mph, and time to 30 mph.

Yamaha Performance Bulletins are generally in the same format as Mercury Marine Boat House Bulletins.

Nautilus, a yacht charter in Croatia provides a performance comparison curve similar to some seen in this paper. For example see the Nautilus curve comparing the Lagoon Power 44 catamaran with the Regal Commodore 4260[14] in **Figure C-1.** The curves show how the Lagoon Power 44 accelerates quicker than the Regal Commodore 4260, which eventually catches up and passes it. If this comparison included one of our virtual boat race images based on the same data, viewers could see how far out front the Lagoon Power 44 was able to get before the Regal Commodore 4260 began to reel the Lagoon back in and when the Commodore eventually passed it.

Stingray has long supplied boat performance test data. Stingray supplies the relevant Yamaha or Mercury performance bulletins along with a radar performance comparison curve[15]. You can select multiple boats and see their curves overlaid upon one another. See **Figure C-2** and **Figure C-3**. Both runs are Yamaha 150 horsepower outboard motor powered Stingray 212SC deck boat models. The red curve has a 17 inch pitch Reliance propeller while the green curve was powered by a 17 inch pitch Vensura propeller. This example visibly illustrates performance changes vs changes made to the vessel (two propeller configurations).

---

[13] 2018 Ranger Z520L Data Run. Pete's Ranger Channel. July 14, 2017. YouTube. https://www.youtube.com/watch?v=1Gg40t6SpN8 viewed 26 September 2020.

[14] Lagoon Power 44 vs Regal Commodore 4260. Nautilus. https://www.nautilus.hr/lagoon_power_44_speed.htm viewed 28 September 2020.

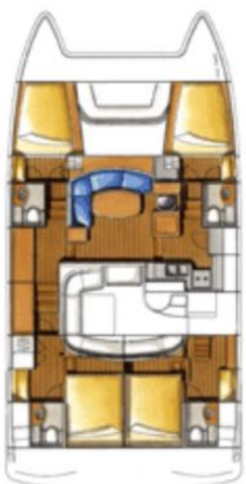
[15] Stingray Radar Performance Comparison page. http://www.stingraypb.com/boat_performance/radar_gun/selectgraphs.php?test_type=&motor= viewed 28 September 2020.

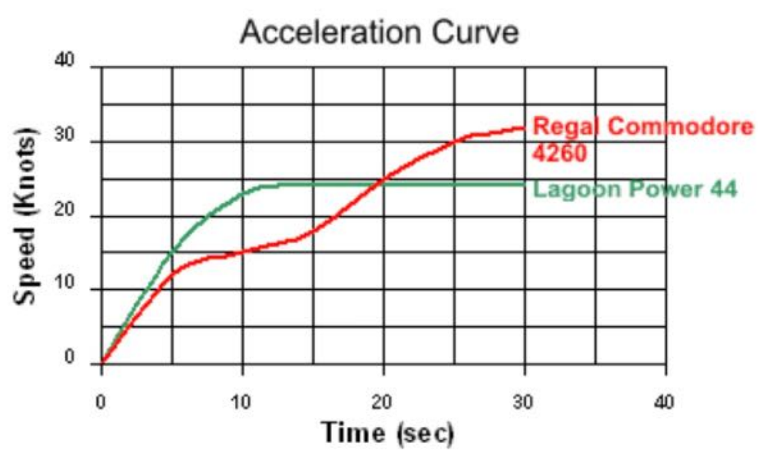
LAGOON POWER 44 VS REGAL COMODORE 4260 - ACCELERATION

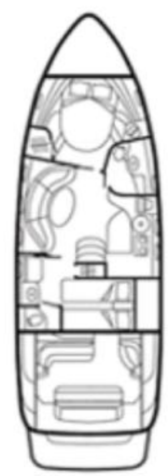
Figure C-1

Acceleration

You probably think that this floating villa accelerate like a turtle, but actually Lagoon Power 44 accelerate better than many monohull powerboats with narrower beam and much stronger engines. The LP 44 boat feels light and responsive. Lagoon Power 44 has the ability to accelerate quickly and jump up when you decide to put the coals to it. Those slim power cat hulls don't have much drag, so throttle response is much quicker than you'd expect on a single-hull boat. Until you get used to this, treat the throttles with respect.







Lagoon Power 44 LOA: 13,04 m

Regal Commodore 4260 LOA: 12,60 m

Lagoon Power, with 2 x 310 HP, achieve its maximum speed of 24 knots in 10 to 12 seconds. In the same time Regal, with 2 x 480 HP, achieve only 15 knots. Regal needs additional 10 seconds to achieve 24 knots.

stingrayboats.com/boat_performance/radar_gun/selectgraphs.php?boat=212S&motor=&testtype=

STINGRAY

Home | Locate a Dealer | Request Brochure | Search

Figure C-2

Radar Performance Comparison

Use this calculator to compare real-time performance graphs on different model, motor, propeller, and load combinations. This real-time data was collected using Stalker Radar connected to a laptop running special software.

Top speed runs are taken with the boat running at full speed toward a stationary radar gun from two directions. With the radar unit set up on a stationary platform and not in the boat, tests have proven to be more accurate and more repeatable.

Acceleration tests are taken with the boat running away from the radar gun and are represented by a curved line starting at zero.

Ski Tests are taken to compare the boat's performance using different propellers pulling a skier. For accuracy and repeatability in the ski test, a certified ski drone is used to simulate the load of an average-weight slalom skier. Ski tests are performed from a standing start in deep water with the boat loaded as indicated in each test.

Tests shown are for information only and may not represent the performance of your particular model boat.

Select a model from the drop-down menu below.

Model: 212S   Type of Test: All   Motor: All

Select the models to compare by checking the box in the left-hand column. Then click the 'Graph' button at the bottom of the page to view the selected tests.

	Model	Motor	Drive	Propeller	RPM	Test Type	PPL	Date	Comments
☑	212SC	Yam 2.7 150hp	OB/2.00	17" Reliance	5800	Top End Run	9	2014-10-27	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Reliance	5800	Acceleration	9	2014-10-27	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Reliance	6000	Top End Run	2	2014-10-27	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Reliance	6000	Acceleration	2	2014-10-27	
☐	212SC	Yam 2.7 150hp	OB/2.00	16" Yam Alm	6100	Top End Run	1	2014-10-23	
☑	212SC	Yam 2.7 150hp	OB/2.00	17" Yam Alm	5800	Acceleration	1	2014-10-23	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Yam Alm	5800	Top End Run	1	2014-10-23	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Turbo	5900	Acceleration	1	2014-10-23	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Turbo	5900	Top End Run	1	2014-10-23	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Vensura	6100	Acceleration	1	2014-10-23	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Vensura	6100	Top End Run	1	2014-10-23	
☐	212SC	Yam 2.7 150hp	OB/2.00	16" Yam Alm	6100	Acceleration	1	2014-10-23	
☐	212SC	Yam 2.7 150hp	OB/2.00	18" Reliance	5800	Top End Run	1	2014-10-23	
☐	212SC	Yam 2.7 150hp	OB/2.00	18" Reliance	5800	Acceleration	1	2014-10-23	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Black Max	6100	Top End Run	1	2014-10-23	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Black Max	6100	Acceleration	1	2014-10-23	
☐	212SC	Yam 2.7 150hp	OB/2.00	17	5400	Top End Run	1	2014-10-15	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Black Max	5800	Acceleration	1	2014-10-15	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Black Max	5800	Top End Run	1	2014-10-15	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Turbo	5600	Acceleration	1	2014-10-15	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Turbo	5600	Top End Run	1	2014-10-15	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Vensura	5700	Acceleration	1	2014-10-15	
☐	212SC	Yam 2.7 150hp	OB/2.00	17" Vensura	5700	Top End Run	1	2014-10-1	

Screenshot

stingrayboats.com/boat_performance/radar_gun/linegraph.php

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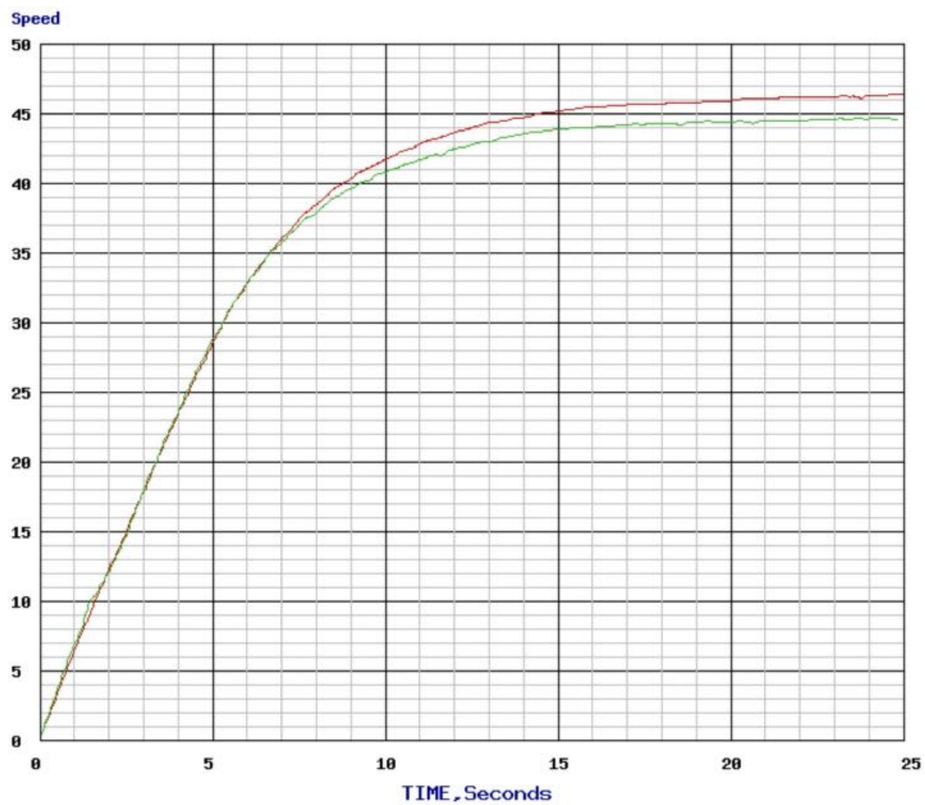
Figure C-3

Stalker Radar Performance Data

The graph below shows the results of the selected tests. Look below the graph for exact speed numbers.



Model	Motor	HP	Prop	RPM	Test Type	Comment	Top Speed	0-20 MPH	0-30 MPH	0-40 MPH
212SC	Yam 2.7	150	17" Reliance	6000	Acceleration		----	3.39	5.34	8.83
212SC	Yam 2.7	150	17" Vensura	6100	Acceleration		----	3.39	5.31	9.31

If Stingray's radar comparison tool included pushing their existing data into one of our virtual boat race charts the reader could see how quickly the Reliance propeller pulled away from the Vensura propeller version.

We encourage all those supplying top speed run data to review this paper, then develop and incorporate any concepts they see fit for their application.

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Note - Boat Coast Down Testing is sometimes spelled as boat coast-down testing or boat coastdown testing.

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